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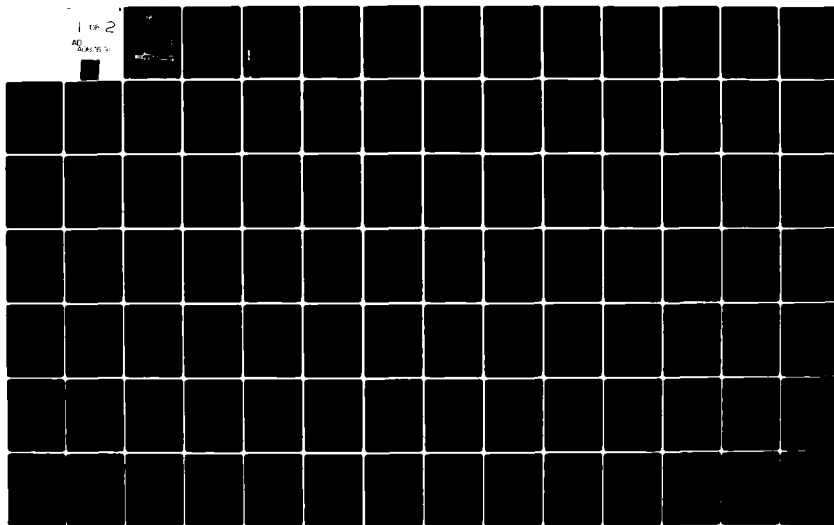
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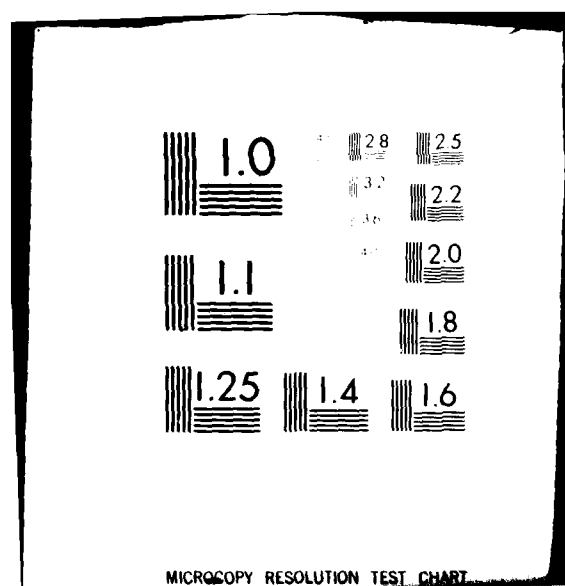
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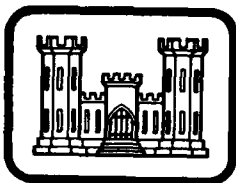
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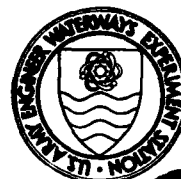
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# LEVEL II



TECHNICAL REPORT SL-80-4

## STRENGTH DESIGN OF REINFORCED CONCRETE HYDRAULIC STRUCTURES

Report I

### PRELIMINARY STRENGTH DESIGN CRITERIA

by

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July 1980

Report I of a Series

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>The overall objective of this study was to develop a realistic strength design methodology for reinforced concrete hydraulic structures and to devise an accurate and efficient design procedure for implementing these strength design methods. The first phase of this study developed preliminary strength design criteria that will yield designs equivalent to those designed by the working-stress method for hydraulic structures. The results of this first phase study are given in this report.<br>(Continued) |                                     |   |

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20. ABSTRACT (Continued).

The preliminary strength design criteria have been developed, including load factors and loading combinations, factored base reactions, design strength for reinforcement, distribution of flexural reinforcement, shrinkage and temperature reinforcement, details of reinforcement, control of deflections, maximum tension reinforcement of flexural members, minimum reinforcement of flexural members, provisions for combined flexure and axial load, and shear strengths of various hydraulic structural members. A comprehensive commentary discussing the considerations and background information used in developing the strength design criteria is also included. Examples for designing typical hydraulic structural members using the strength design method are given. A preliminary study on the use of the probabilistic approach to derive the load factors and strength reduction factors for hydraulic structures is presented.

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## PREFACE

The study reported herein was conducted in the Structures Laboratory (SL), U. S. Army Engineer Waterways Experiment Station (WES), under the sponsorship of the Office, Chief of Engineers (OCE), U. S. Army, as a part of Civil Works Investigation Work Unit 31623. Mr. Donald R. Dressler of the Structures Branch, Engineering Division, OCE, served as technical monitor.

This study was conducted during the period October 1978 to September 1979 under the general supervision of Mr. Bryant Mather, Acting Chief, SL; Mr. John Scanlon, Chief, Engineering Mechanics Division, SL; and Mr. James E. McDonald, Chief, Structures Branch, SL. This study was conducted and this report was prepared by Dr. Tony C. Liu, SL.

The assistance and cooperation of many persons were instrumental in the successful completion of this study. The author wishes to acknowledge Mr. Donald R. Dressler, OCE; Professor Phil M. Ferguson, University of Texas at Austin; Mr. Ervell A. Staab, Missouri River Division; Mr. Chester F. Berryhill, Southwestern Division; Mr. V. M. Agostinelli, Lower Mississippi Valley Division; Mr. Garland E. Young, Fort Worth District; Mr. Marion M. Harter, Kansas City District; Mr. George Henson, Tulsa District; and Dr. Paul Mlakar, Dr. N. Radhakrishnan, and Mr. William A. Price, WES, for their critical review of the manuscript.

The Commanders and Directors of the WES during this study and the preparation and publication of this report were COL John L. Cannon, CE, and COL Nelson P. Conover, CE. The Technical Director was Mr. F. R. Brown.

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CONVERSION FACTORS, INCH-POUND TO METRIC (SI)  
UNITS OF MEASUREMENT

Inch-pound units of measurement used in this report can be converted to metric (SI) units as follows:

| <u>Multiply</u>                | <u>By</u>   | <u>To Obtain</u>             |
|--------------------------------|-------------|------------------------------|
| feet                           | 0.3048      | metres                       |
| inches                         | 25.4        | millimetres                  |
| kips (force)                   | 4448.222    | newtons                      |
| kips (force) per foot          | 1459.3904   | newtons per metre            |
| kips (force) per inch          | 175126.8    | newtons per metre            |
| kips (force) per square foot   | 47.88026    | kilopascals                  |
| kips (force) per square inch   | 6.894757    | megapascals                  |
| kip (force)-feet               | 1355.818    | newton-metres                |
| kip (force)-inches             | 112.9848    | newton-metres                |
| pounds (force)                 | 4.448222    | newtons                      |
| pounds (force) per square inch | 0.006894757 | megapascals                  |
| pounds (mass) per cubic foot   | 16.01846    | kilograms per cubic<br>metre |
| square inches                  | 6.4516      | square centimetres           |

## STRENGTH DESIGN OF REINFORCED CONCRETE

### HYDRAULIC STRUCTURES

#### PRELIMINARY STRENGTH DESIGN CRITERIA

#### PART I: INTRODUCTION

##### Background

1. Since 1963 the structural engineering profession has been gradually adopting the strength design (SD) approach in lieu of the working-stress method that is the basis of current Engineer Manual EM 1110-1-2101, "Working Stresses for Structural Design" (Office, Chief of Engineers 1963). This EM permits the use of the SD method, but does not provide adequate guidance for proportioning structural members for strength and service requirements. The SD criteria for reinforced concrete hydraulic structures (RCHS) need to be developed for several reasons:

- a. The American Concrete Institute (ACI) Building Code and the American Association of State Highway and Transportation Officials (AASHTO) Bridge Code are not directly applicable to hydraulic structures.
- b. The SD approach is more realistic and is potentially capable of producing more economical structures without compromising safety requirements (Winter and Nilson 1972).
- c. Structural engineering research in the United States and abroad will only be updated in terms of the SD approach.
- d. Engineering schools are only emphasizing the SD approach in their design courses; recent and future engineering graduates will not be familiar with the working-stress method.

2. Consequently, a comprehensive study was initiated at WES in 1978 to develop a realistic SD methodology for RCHS and to devise an accurate and efficient design procedure for implementing these SD methods.

##### Objective

3. The purpose of the first phase of study was to develop general

SD criteria that will yield designs (i.e. member dimensions and reinforcements) equivalent to those designed by the working-stress method for RCHS. The results of this first phase study are reported herein. The second phase of this study, which will be initiated in FY 80, will develop a realistic SD methodology and practical procedure that accounts for the special loading and service characteristics of particular RCHS.

## PART II: STRENGTH DESIGN CRITERIA

### Introduction

4. This part defines the strength design criteria for reinforced concrete hydraulic structures. The considerations and background information used in developing these criteria are given in Part III.\* Examples for designing typical hydraulic structural members using the strength design method are given in Appendix A.

5. A hydraulic structure is defined as a structure that will be subjected to submergence, wave action, spray, chemically contaminated atmosphere, and severe climatic conditions (OCE 1963). Typical hydraulic structures are stilling basin slabs and walls, concrete-lined channels, portions of powerhouses, spillway piers, spray walls and training walls, flood walls, intake and outlet structures below maximum high water and wave action, lock walls, guide and guard walls, and retaining walls subject to contact with water (OCE 1963).

6. Reinforced concrete hydraulic structures may be designed with the strength design method in accordance with the current "Building Code Requirements for Reinforced Concrete," ACI 318-77\*\* (ACI 1977a) except as hereinafter specified. The notations used are the same as those used in ACI 318-77, except those defined in this report.

### Strength Requirements

#### Required strength

7. Reinforced concrete hydraulic structures and structural members shall be designed to have design strengths at all sections at least equal to required strengths calculated for the factored loads and forces in the following combinations that are applicable:†

---

\* Part III is a commentary on the criteria.

\*\* Hereinafter referred to as ACI 318-77.

† Both the full and zero values of  $L$  in Equations 3 and 4 shall be considered to determine the more severe condition.

$$U = 1.5D + 1.9 (L + H_w + H_p + F_w + F_p + F_u) \quad (1)$$

$$U = 0.9D + 1.9 (H_w + H_p + F_w + F_p + F_u) \quad (2)$$

$$U = 0.75 [1.5D + 1.9 (L + H_w + H_p + F_w + F_p + F_u + P + W + T)] \quad (3)$$

$$U = 0.75 [1.5D + 1.9 (L + H_w + H_p + F_w + F_p + F_u + P + E + T)] \quad (4)$$

where

- $H_w$  = earth mass, or related internal moments and forces
- $H_p$  = lateral earth pressure, or related internal moments and forces
- $F_w$  = water mass, or related moments and forces
- $F_p$  = lateral water pressure or related internal moments and forces
- $F_u$  = vertical uplift pressure or related internal moments and forces
- $P$  = additional pressure due to wave action

Base reactions for hydraulic structures

8. The factored base reactions for most hydraulic structures shall be approximated by applying a load factor of 1.5 to 1.9 to the base reactions obtained from a stability analysis for unfactored normal load cases. Factored base reactions for abnormal load cases shall be approximated by applying reduced load factors to the base reactions from appropriate stability analyses, consistent with paragraph 7.

Design strength for reinforcement

9. Design shall be based on a 40,000-psi yield strength of reinforcement for Grades 40 and 60 steel (American Society for Testing and Materials 1978). The reinforcement with yield strength in excess of Grade 60 shall not be used, except for prestressing tendons.

## Serviceability Requirements

### Distribution of flexural reinforcement

10. For reinforced concrete hydraulic structures, spacing of flexural tension reinforcement shall not generally exceed 12 in.\*

11. The spacing of flexural tension reinforcement exceeding the limit specified in paragraph 9, but less than 18 in. may be used if it can be justified. In no case shall the flexural tension reinforcement spacing exceed 18 in.

### Shrinkage and temperature reinforcement

12. Area and spacing of shrinkage and temperature reinforcement shall be in accordance with the requirements specified in EM 1110-2-2103 (OCE 1971), or in appropriate Engineering Manuals for specific structures.

### Details of reinforcement

13. Bending and splicing of reinforcement, minimum reinforcement spacing, and minimum concrete cover for principal reinforcement shall be in accordance with the requirements specified in EM 1110-2-2103 (OCE 1971).

### Control of deflections

14. Deflections at service loads need not be computed if the limits of the reinforcement ratio specified in paragraph 16 are not exceeded.

15. For reinforcement ratios exceeding the limits specified in paragraph 16, deflections shall be computed in accordance with Section 9.5 of ACI 318-77, or other methods that predict deflections in substantial agreement with the results of comprehensive tests.

---

\* A table of factors for converting inch-pound units of measurement to metric (SI) units is given page 3.

### Flexure and Axial Loads

#### Maximum tension reinforcement of flexural members

16. For flexural members, and for members subjected to combined flexure and compressive axial load when the design axial load strength  $\phi P_n$  is less than the smaller of either  $0.10 f'_c A_g$  or  $\phi P_b$ , the ratio of tension reinforcement  $\rho$  provided shall not generally exceed  $0.25 \rho_b$ .\*

17. Reinforcement ratios exceeding the limits specified in paragraph 16 but less than  $0.50 \rho_b$  may be used if deflections are not shown to significantly affect the operational characteristics of the structure.

18. Reinforcement ratios in excess of  $0.50 \rho_b$  shall not be used unless a detailed investigation (e.g. laboratory testing, linear or nonlinear finite element analyses) of serviceability requirements, including computation of deflections, is conducted in consultation with higher authority.

#### Minimum reinforcement of flexural members

19. At any section of a flexural member where tension reinforcement is required by analysis, the minimum reinforcement requirements specified in ACI 318-77 shall apply except that the  $f_y$  shall be in accordance with paragraph 9.

#### Combined flexure and axial load

20. The design axial load strength  $\phi P_n$  of compression member shall not be taken greater than the following:

$$\phi P_{n(max)} = 0.70 \phi A_g (0.85 f'_c + f_y \rho_g) \quad (5)$$

---

\* The reinforcement ratio for conduits or culverts, designed in accordance with EM 1110-2-2902 (OCE 1969), shall not generally exceed  $0.375 \rho_b$ .

where  $\rho_g$  is the ratio of area of reinforcement to the gross concrete area,  $A_g$ .

21. The strength of a cross section is controlled by compression if the factored axial load  $P_u$  has an eccentricity  $e$  no greater than that given by Equation 6, and by tension if  $e$  exceeds this value.

$$e_b = \frac{\rho' m (d - d') + 0.1 d}{(\rho' - \rho) m + 0.6} \quad (6)$$

where

$$m = \frac{f_y}{0.85 f'_c}$$

22. Sections controlled by compression shall be proportioned by Equation 7.\*

$$\frac{\phi P_n}{P_A} + \frac{\phi M_n}{M_F} \leq 1 \quad (7)$$

where

$$P_A = 0.65 (1 + \rho_g m) f'_c A_g$$

$$M_F = 0.11 f'_c b h^2$$

23. For sections controlled by tension, the moment strength  $\phi M_n$  shall be considered to vary linearly with the axial load strength  $\phi P_n$  from  $\phi M_o$  (when the section is in pure flexure) to  $\phi M_b$  (when the axial load strength is equal to  $\phi P_b$ );  $\phi M_b$  and  $\phi P_b$  shall be determined from  $e_b$  and Equation 7);  $\phi M_o$  from Equation 8.

$$\phi M_o = \phi \rho f_y b d^2 \left( 1 - 0.59 \rho \frac{f_y}{f'_c} \right) \quad (8)$$

---

\* For conduits or culverts,  $M_F = 0.14 f'_c b h^2$ .



### Shear Strength Requirements

24. The nominal shear strength  $V_c$  provided by concrete shall be computed in accordance with ACI 318-77 except as modified by paragraphs 26-28.

25. Provisions of paragraphs 26 and 27 shall apply to straight members as follows:

- a. Members with an ultimate shear strength limited to the load capacity that causes formation of the first inclined crack.
- b. Members with beams or frames having rigid, continuous joints or corners.
- c. Members subjected to uniformly distributed loads, or loads closely approximating this condition.
- d. Members subjected to internal shear, flexure, and axial compression, but not axial tension.
- e. Members with rectangular cross-sectional shapes.
- f. Members with  $\ell_n/d$  less than 10 and  $f'_c$  not more than 6000 psi.
- g. All straight members designed in accordance with paragraphs 26 and 27 shall include the stiffening effects of wide supports and haunches in determining moments, shears, and member properties.
- h. The reinforcing details for all straight members designed in accordance with paragraphs 26 and 27 shall meet the following requirements:
  - (1) Straight, full-length reinforcement shall be used. Flexural reinforcement shall not be terminated even though it is no longer a theoretical requirement.
  - (2) Reinforcement in the exterior face shall be bent around corners and shall have a vertical lap splice in a region of compressive stress.
  - (3) Reinforcement in the interior face shall extend into and through the supports.
- i. Shear strength for straight members shall not be taken greater than  $20\sqrt{f'_c}bd$  when  $\ell_n/d$  is between 2 and 10

$$V_c = 2 \left( 12 - \frac{\ell_n}{d} \right) \sqrt{f'_c} bd \quad (9)$$

26. At a distance  $0.15 \ell_n$  from the face of the support, for straight members with  $\ell_n/d$  between 2 and 6

$$V_c = \left[ \left( 11.5 - \frac{\ell_n}{d} \right) \sqrt{f'_c} \sqrt{1 + \frac{\frac{N_u}{A_g}}{5 \sqrt{f'_c}}} \right] bd \quad (10)$$

27. At points of contraflexure, for straight members with  $\ell_n/d$  between 6 and 10

$$V_c = \left[ 11,000 \frac{(0.046 + \rho) \left( 12 + \frac{N_u}{V_u} \right)}{\left( 19 + \frac{\ell'}{d} \right)} \sqrt{\frac{f'_c}{4,000}} \right] bd \quad (11)$$

The length  $\ell'$  is the distance between the points of contraflexure.

28. At points of maximum shear, for uniformly loaded curved cast-in-place members with  $R/d > 2.25$  where  $R$  is the radius of curvature to the centerline of the member

$$V_c = \left[ l_4 \sqrt{f'_c} \sqrt{1 + \frac{\frac{N_u}{A_g}}{l_4 \sqrt{f'_c}}} \right] bd \quad (12)$$

but the shear strength shall not exceed  $10 \sqrt{f'_c} bd$ .

29. Shear strength based on the results of detailed model tests approved by higher authority shall be considered a valid extension of the provisions in paragraphs 26-28.

### PART III: COMMENTARY

30. This part discusses some of the considerations and background information used in developing the strength design criteria contained in Part II.

31. As an alternative to the working-stress design method for reinforced concrete hydraulic structures, the design provisions of ACI 318-77 (ACI 1977) are generally applicable. However, because of the unique strength and serviceability requirements of the RCHS, some design criteria of ACI 318-77 need to be modified. The specific design criteria that are different from those of ACI 318-77 are given in Part II. A comparison of the design criteria for RCHS and ACI 318-77 is presented in Appendix B. Most of the notations used in Part II are the same as those used in ACI 318-77 and therefore are not defined. Special notations that are not used in the ACI 318-77 have been defined in Part II.

#### Strength Requirements

##### Required strength

32. The dead load factor of ACI 318-77 is increased from 1.4 to 1.5 to account for the greater load uncertainties for hydraulic structures. The live load factor is increased from 1.7 to 1.9. The basis for this modification is discussed in subsequent paragraphs.

33. Considering the loading combination of live and dead loads, the required capacity of a flexural member shall be at least equal to

$$M_u = K_D M_D + K_L M_L \quad (13)$$

where

- $M_u$  = required factored moment capacity
- $K_D$  = dead load factor
- $M_D$  = bending moment due to dead load
- $K_L$  = live load factor
- $M_L$  = bending moment due to live load

34. Since designs using the working-stress design method specified in EM 1110-1-2101 (OCE 1963) are satisfactory, the strength design method should yield equivalent designs. Therefore

$$M_u = (L_F)M_w \quad (14)$$

where

$L_F$  = overall load factor

$M_w$  = required moment capacity derived from working-stress method

Substituting Equation 14 into Equation 13

$$(L_F)M_w = K_D M_D + K_L M_L \quad (15)$$

35. Recent studies indicated that the overall load factor is approximately 1.9 for  $\rho < 0.25 \rho_b$  and  $\phi = 0.90$  (Figures 1 and 2).

Substituting 1.9 for  $L_F$  into Equation 15

$$1.9 M_w = K_D M_D + K_L M_L \quad (16)$$

Since  $M_w = M_D + M_L$

then

$$1.9 = K_D \frac{M_D}{M_D + M_L} + K_L \frac{M_L}{M_D + M_L} \quad (17)$$

$$= K_D \frac{\frac{M_D}{M_L}}{\frac{M_D}{M_L} + 1} + K_L \frac{1}{\frac{M_D}{M_L} + 1}$$

Let

$$\frac{M_D}{M_L} = \alpha$$

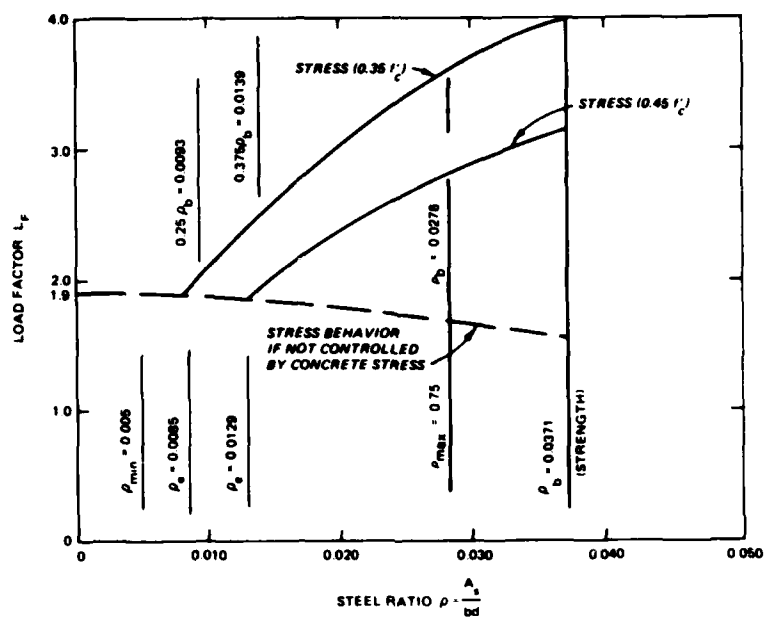


Figure 1. Overall load factor-steel ratio,  
 $f_y = 40,000$  psi and  $f'_c = 3,000$  psi

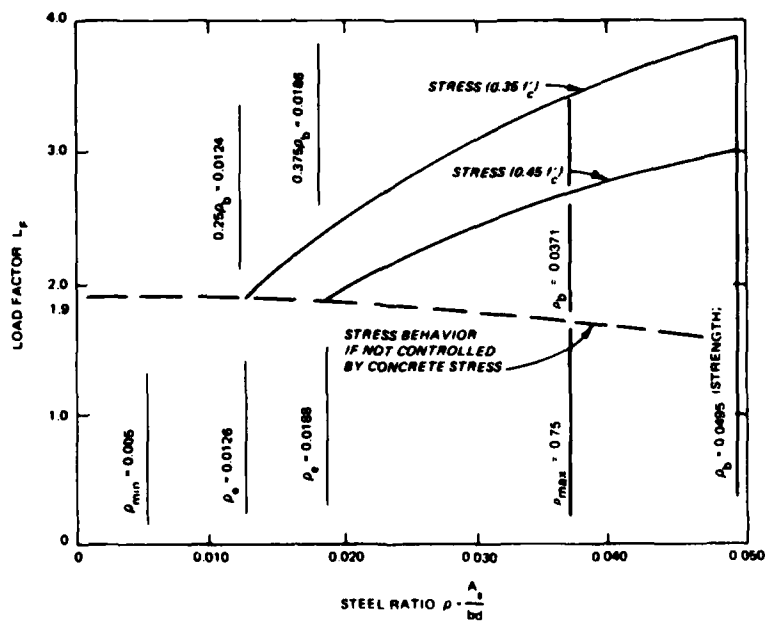


Figure 2. Overall load factor-steel ratio,  
 $f_y = 40,000$  psi and  $f'_c = 4,000$  psi

and

$$1.9 = K_D \frac{\alpha}{1 + \alpha} + K_L \frac{1}{1 + \alpha} \quad (18)$$

or

$$K_L = 1.9 (1 + \alpha) - \alpha K_D \quad (19)$$

Since

$$K_D = 1.5$$

therefore

$$K_L = 1.9 + 0.4\alpha \quad (20)$$

It can be seen for a given  $K_D$  that the value of  $K_L$  is not a constant but a function of  $\alpha$  (Figure 3).

36. For many hydraulic structures, the dead load of the concrete structure is much smaller than the live load:  $\alpha \ll 1$ . To be consistent with the concept of constant live load factor used in ACI 318-77, the value  $\alpha$  is assumed to be zero; and therefore

$$K_L = 1.9 \quad (21)$$

In special cases where the dead load effects are significant, a larger  $K_L$  shall be used, and in these cases  $K_L$  should be determined from Equation 20.

37. In paragraph 7, a load factor of 1.9 was chosen for all loadings due to lateral earth pressure, earth weight, lateral water pressure, vertical uplift pressure, water weight, wave pressure, wind loads,

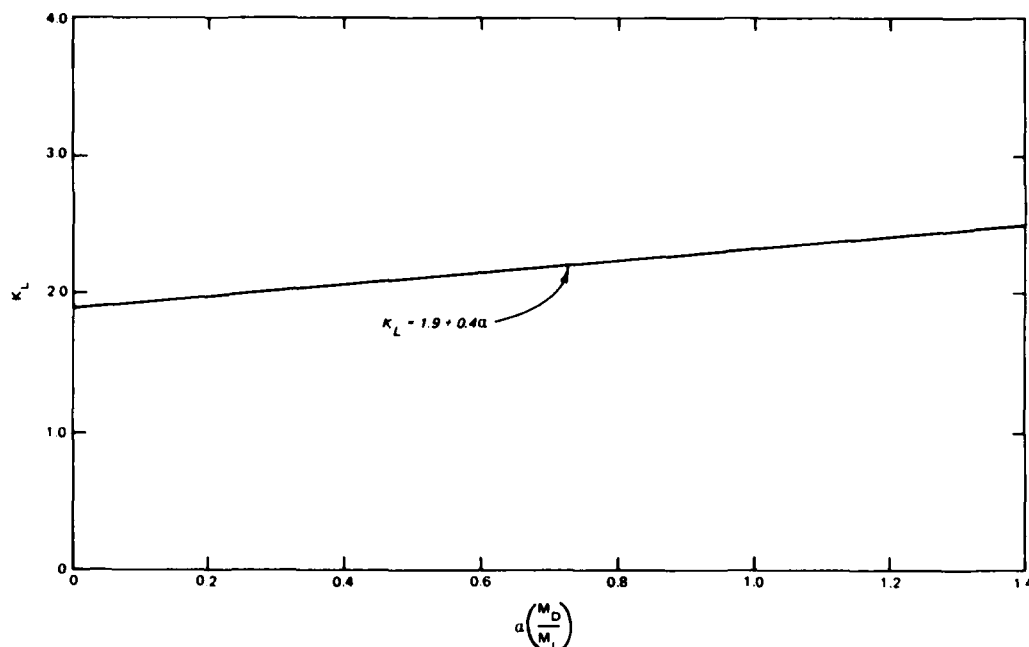


Figure 3. Relation between  $K_L$  and  $\alpha(K_D = 1.5)$

earthquake effects, and the effects of temperature, creep, shrinkage, and differential settlement. Load factors, depending upon the degree of uncertainty of loads considered can be determined using a probabilistic approach (Appendix C) and will be refined during future phases of this study.

38. The reduction factor of 0.75 used in Equations 3 and 4 is consistent with the provision of increasing the allowable stresses by 33-1/3 percent for Group II loadings (OCE 1963) for the working-stress designs.

#### Base reactions for hydraulic structures

39. Theoretically, the base reactions for hydraulic structures should be evaluated from the factored loads such as earth pressures, water pressures, and structural weights. However, since the bearing pressures are caused by loadings with different load factors, such a procedure will cause relocation of the resulting eccentricities and lead to base reactions that are in principle different from those obtained

under service load conditions. For the purposes of design, paragraph 8 specifies that a load factor between 1.5 and 1.9 be applied to the base reactions obtained from the investigation of the service load conditions. A weighted average method, as given in Equation 22, may be used for determining the load factor for base reaction for strength design:

$$K_B = \frac{K_D |D| + K_{Hw} |H_w| + K_{Hp} |H_p| + K_{Fw} |F_w| + K_{Fp} |F_p| + K_{Fu} |F_u|}{|D| + |H_w| + |H_p| + |F_w| + |F_p| + |F_u|} \quad (22)$$

where

- $K_B$  = load factor for bearing pressure
- $K_D$  = load factor for dead load
- $K_{Hw}$  = load factor for earth weight
- $K_{Hp}$  = load factor for lateral earth pressure
- $K_{Fw}$  = load factor for water weight
- $K_{Fp}$  = load factor for lateral water pressure
- $K_{Fu}$  = load factor for uplift pressure
- $| |$  = absolute value of the load considered

Justification for selecting an appropriate load factor should be submitted to the higher authority.

#### Design strength for reinforcement

40. Paragraph 9 limits the yield strength of reinforcement for design to 40,000 psi. This limit is more restrictive than ACI 318-77, which placed the upper limit at 80,000 psi. Since the maximum width of crack due to load is approximately proportional to stress in reinforcement, this limit is necessary to minimize the crack width in the hydraulic structures.

#### Serviceability Requirements

##### Distribution of flexural reinforcement

41. Control of cracking in RCHS is particularly important. RCHS



designed by the working-stress method have low concrete and steel stresses and have served their intended functions with very limited flexural cracking.

42. The ACI 318-77 criteria for distribution of flexural reinforcement in beams and one-way slabs is based on the results of small beam tests (Gergely and Lutz 1968). The present ACI criteria may not be applicable for large RCHS members.

43. Extensive investigation of the cracking phenomenon in reinforced concrete structures revealed that smaller bar sizes placed at closer spacings are more effective in controlling flexural cracking than a few larger bars of equivalent area placed at wider spacing between bars (ACI 1972). The best crack control is obtained when the reinforcement is well distributed over the zone of maximum concrete tension. The provisions of paragraph 10 limiting the bar spacing to 12 in. are empirical but have been used satisfactorily for hydraulic structures.

#### Shrinkage and temperature reinforcement

44. Volume changes in concrete due to drying shrinkage, heat of hydration of cement, and seasonal variations in temperature in restrained members can produce excessive tensile stresses and thus cause wide cracks. Therefore, shrinkage and temperature reinforcement is required at right angles to the principal reinforcement to prevent excessive cracking.

45. EM 1110-2-2103 (OCE 1971) classifies three degrees of restraint for concrete hydraulic structural members: unrestrained, restrained, or partially restrained. The areas of reinforcement required vary from 0.2 percent to 0.4 percent depending on the degree of restraint. The shrinkage and temperature reinforcement requirements for conduits or culverts given in EM 1110-2-2902 (OCE 1969) shall be satisfied.

#### Details of reinforcement

46. According to EM 1110-2-2103 (OCE 1971), the bending and splicing of reinforcement shall conform to ACI 318-77.

47. The maximum size of coarse aggregate used in hydraulic structures is generally larger than that used in building structures.

Therefore, the bar spacing requirements specified in EM 1110-2-2103 (OCE 1971) are generally larger than those of ACI 318-77.

48. Thick concrete covers (from 4 to 6 in.) for hydraulic structures are generally required not only for protection of reinforcement against corrosion but also for erosion protection against abrasion and cavitation.

#### Control of deflections

49. The members designed by strength-design methods are often more slender than those designed by working-stress methods. Reasons for this include (a) the use of large steel ratios and (b) the use of high-strength steel and concrete. Because the deflection of slender members may in some cases exceed desirable limits, paragraph 15 specifies that deflection shall be checked when the reinforcement ratio exceeds  $0.25\rho_b$  (or  $0.375\rho_b$  for conduits or culverts).

#### Flexure and Axial Loads

##### Maximum reinforcement of flexural members

50. The limitation on the amount of tensile reinforcement that may be used in a flexural member is to ensure that flexural members designed by the strength criteria will have ductile behavior.

51. The maximum tension reinforcement of flexural members for hydraulic structures is limited to  $0.25\rho_b$  and to  $0.375\rho_b$  for conduits or culverts. The derivation of this limitation is given in Appendix D. This limit is much smaller than  $0.75\rho_b$  allowed in ACI 318-77. The low reinforcement limit will result in a stiffer structure, which is considered desirable for hydraulic structures.

52. For unusual situations when a larger amount of reinforcement is necessary because of physical constraints on member dimensions due to functional or esthetic requirements, paragraph 17 limits the reinforcement ratio to  $0.50\rho_b$  provided that the deflection criteria specified in paragraphs 14 and 15 are met.

Minimum reinforcement of flexural members

53. This provision applies to members that for functional or other reasons are much larger in cross section than required by strength consideration. The computed moment strength as a reinforced concrete section with very little tensile reinforcement becomes less than that of the corresponding plain concrete section computed from its modulus of rupture. Failure in such a case could be quite sudden. A minimum percentage of reinforcement should be provided to prevent such a mode of failure.

Combined flexural and axial load

54. The strength design procedure for members subjected to combined flexure and axial load can be summarized in Figure 4.

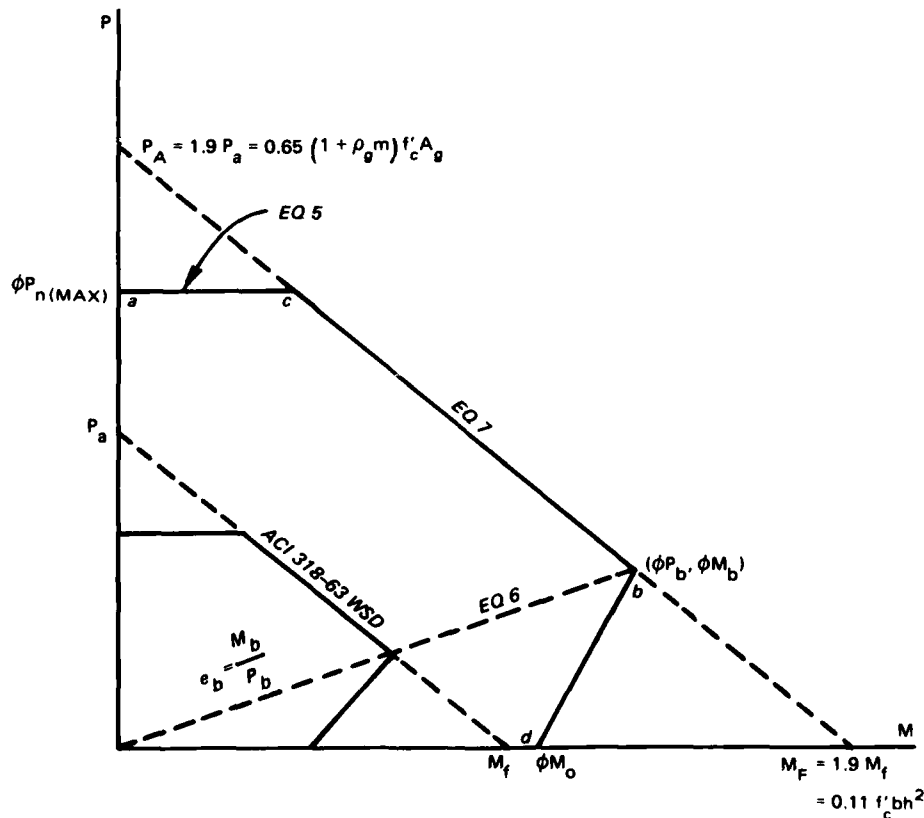


Figure 4. Typical interaction diagram

55. For zero or small eccentricities (interval  $ac$ ), the strength of a section is that for concentric compression (Equation 5). Equation 5 is derived to yield  $\phi P_{n(max)}$  approximately 1.9 times the maximum allowable axial load calculated by the working-stress method using Section 1403 of ACI 318-63. The  $\phi P_{n(max)}$  calculated by Equation 5 is about 10 percent less than that computed by Equation (10-2) of ACI 318-77.

56. For moderate eccentricities, when compression governs in the interval  $cb$ , the interaction Equation 7 applies. It is represented by the straight line connecting  $P_A$  (when  $M = 0$ ) and  $M_F$  (when  $P = 0$ ). In that range, one determines  $\phi P_n$  and  $\phi M_n$  from Equation 7 and compares  $\phi P_n$  with  $\phi P_{n(max)}$  as calculated from Equation 5. The smaller of the two is the design strength.

57. Point  $b$  determines the boundary between members governed by compression and those governed by tension. It can be determined by calculating  $e_b$  from Equation 6 and then  $\phi P_b$  from Equation 7.

58. For large eccentricities, when tension governs, a linear variation is assumed between the moment  $\phi M_b$  at the balance point  $b$  and  $\phi M_o$  for simple flexure, shown by straight line  $bd$ .

59. It can be seen from Figure 4 that the general shape of the figure  $acbd$  is very similar to the interaction diagram for working-stress design specified in ACI 318-63 (ACI 1963). The similarity of the figures comes from the fact that the SD criteria are devised to provide comparable design with the conventional working-stress design methods.

### Shear Strength Requirements

#### Shear strength provided by concrete

60. In general, for retaining walls or flood walls, the  $V_c$  shall be computed in accordance with Section 11.3.2 of ACI 318-77. The maximum shear for horseshoe-shaped conduits or culverts will generally occur in the base slab near the walls at a point of high moment, shear, and axial thrust. In this case, the  $V_c$  shall also be computed in accordance with the provisions of Section 11.3.2 of ACI 318-77. For thick

horseshoe-shaped conduits or culverts with  $t/d$  less than 5, the special provisions for deep flexural members given in Section 11.8 of ACI 318-77 may be used, if appropriate. Paragraphs 26 and 27 shall apply.

61. The walls of an intake structure generally have high uniform loads caused by either water pressure or earth embankment fill that result in high axial compression thrusts on the members. Section 11.3 of ACI 318-77 shall be used to compute  $V_c$ . When the structural conditions of intake structure walls are clearly similar to the box conduits investigated in the laboratory (Diaz de Cossio and Siess 1969 and Ruzicka et al. 1976), Equations 10 and 11 may be used. The special provisions for deep flexural members given in ACI 318-77 may apply to some intake structure walls; however, since there is no provision for axial loads, the results will usually be too conservative. Paragraph 28 is generally applicable to uniformly loaded circular or oblong conduits or culverts.

62. For hydraulic structures, the factored shear force  $V_u$  will not generally exceed the shear strength provided by concrete  $\phi V_c$ ; and therefore, shear reinforcement is not normally required. However, in certain areas where  $V_u$  does exceed  $\phi V_c$ , shear reinforcement shall be provided in accordance with Section 11.5.6 of ACI 318-77.

63. The limitations for using Equations 10 and 11 are specified in paragraph 25. These requirements are necessary to ensure that the actual structural performance is consistent with the modes of failure observed in comprehensive tests performed at the University of Illinois by Diaz de Cossio and Siess (1969), Gamble (1977), and Ruzicka (Ruzicka et al. 1976). Special attention should be given to the following:

- a. In frames of normal proportions, the supporting member can be idealized as a line structure in determining moments and shears. However, as the widths of the supporting members become larger relative to the span (e.g., in a thick-walled multiple opening conduit), it becomes necessary to take these widths into account if the distribution of moments is to be adequately predicted. As the supports become wider, one must also reconsider the definition of the span (clear span or center-to-center span), and the effects of the growing joint areas on the flexural stiffness factors. The recommended idealization of thick box

conduits with and without haunches can be found in Diaz de Cossio (1969) and Gamble (1977).

- b. The reinforcing details, including the lengths of bars, the locations of bar terminations, and the locations of bends should be considered both to ensure the adequate behavior of the structure and to minimize material and fabrication costs. The outside reinforcement should be bent around the corners in a thick-walled multiple opening conduit, and the vertical bars should then be lap-spliced in accordance with ACI 318-77. All other bars are straight and full length. No bars should be cut off short, even though in some cases the shape of the bending moment diagrams might appear to allow this to be done. The extensions of the inside bars (bottom bars in top member, top bars in bottom member, and inside bars in exterior vertical members) into and nearly through the supports are recommended.

64. The bent bars have been used in some box conduits, but these become quite inefficient and ineffective when members with small  $\ell_n/d$  values are considered. The use of all straight bars rather than bent bars is recommended for thick-walled conduits.

65. For thick-walled reinforced concrete box conduits or culverts, the lower bound of the nominal shear strength given in Equation 10 is obtained from Ruzicka et al. (1976). Equation 10 is only applicable when the technical requirements specified in paragraph 24 are met.

66. Equation 11 is obtained from Diaz de Cossio and Siess (1969), and is generally applicable to box conduits or culverts with  $\ell_n/d$  between 6 and 10. The requirements specified in paragraph 25 should also be met. For box conduit sections of ordinary dimensions, the critical member for shear is usually the horizontal member.

67. Equation 12 is derived based on a principal stress analysis of an elastic member subjected to axial compression and shear, and in which it is assumed that failure will occur when the principal tensile stress reaches a limiting value of  $\sqrt{f'_c}$ . Equation 12 is only applicable at points of zero bending moment. The nominal shear strength for other sections subjected to combined shear, flexure, and axial thrust should be computed in accordance with the appropriate sections of Chapter 11 of ACI 318-77.

68. In general, the  $R/d$  of the circular or oblong conduits is

greater than 2.25. For unusually thick curved members with  $R/d \leq 2.25$  , the effects of member curvature on the stress distributions should be considered.

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## APPENDIX A: DESIGN EXAMPLES

### Design of Retaining Walls

#### Design data

1. Design information for retaining walls is presented in the following paragraphs.

##### a. Soil data.

Unit mass of sand (submerged) = 65.5 lb/cu ft  
Unit mass of clay (submerged) = 57.5 lb/cu ft  
Active earth pressure coefficient for sand = 0.33  
Passive earth pressure coefficient for clay = 2.04

##### b. Material data.

Concrete compressive strength,  $f'_c = 3,000$  psi  
Yield strength of steel,  $f_y = 40,000$  psi

#### Loading diagrams and dimensions

2. The critical loading diagrams and structural dimensions resulting from stability analysis of service load condition are shown in Figure A1.

#### Design of stem

3. Working-stress design. Referring to Figure A2, the moment at the base of the stem is

$$M = (P_1 \times 8.5) + (P_2 \times 8.5) - (P_3 \times 1.5)$$

where

$$\begin{aligned} P_1 &= \frac{1}{2} \times 0.0625 \times (25.5)^2 \\ &= 20.3 \text{ kips} \end{aligned}$$

$$\begin{aligned} P_2 &= \frac{1}{2} \times 0.33 \times 0.0655 \times (25.5)^2 \\ &= 7.0 \text{ kips} \end{aligned}$$

$$\begin{aligned} P_3 &= \frac{1}{2} \times 0.0625 \times (4.5)^2 \\ &= 0.6 \text{ kips} \end{aligned}$$

Thus

$$\begin{aligned} M &= (20.3 \times 8.5) + (7.0 \times 8.5) - (0.6 \times 1.5) \\ &= 231.2 \text{ kip-ft} \end{aligned}$$

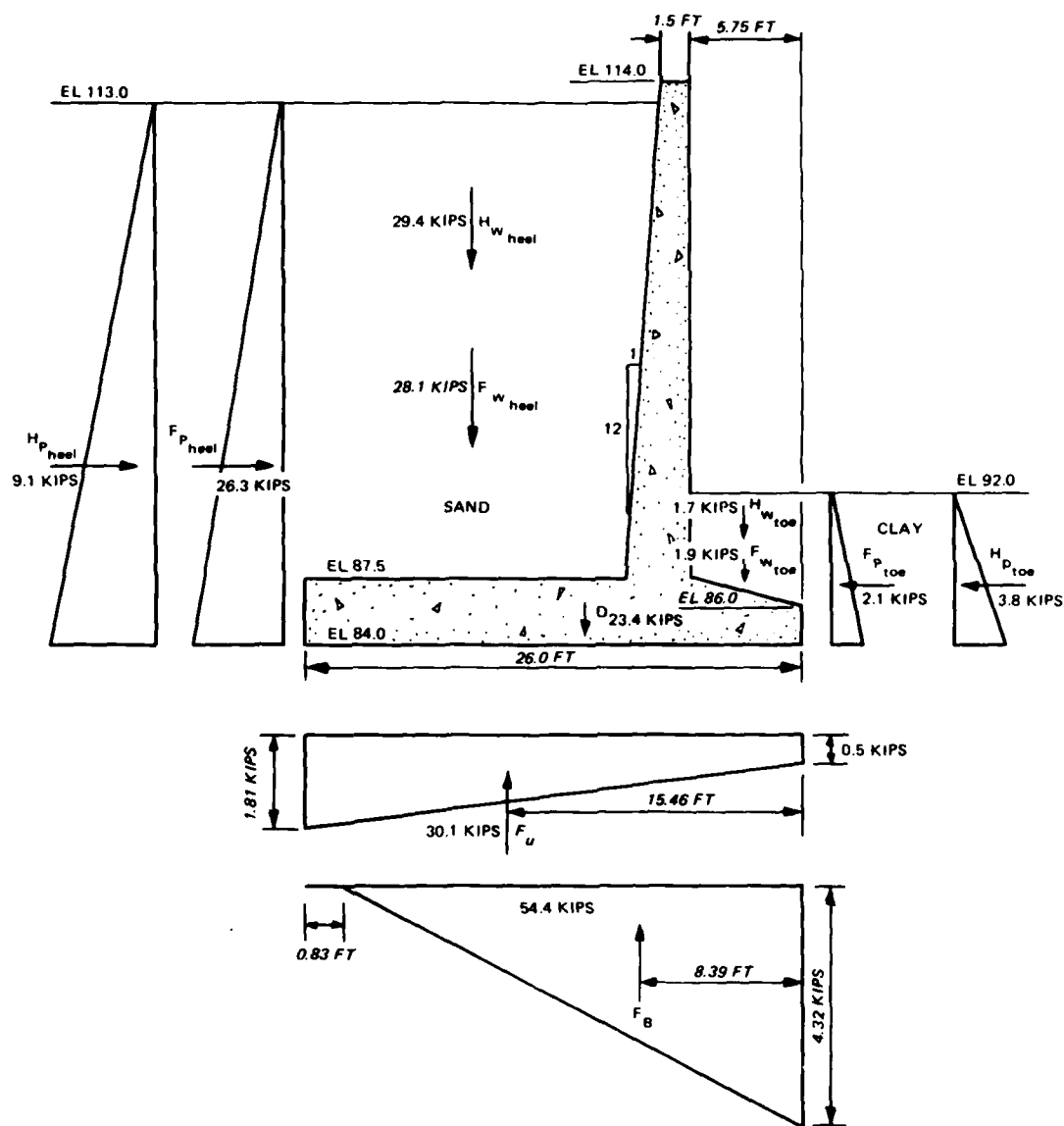


Figure A1. Critical loading diagrams and structural dimensions

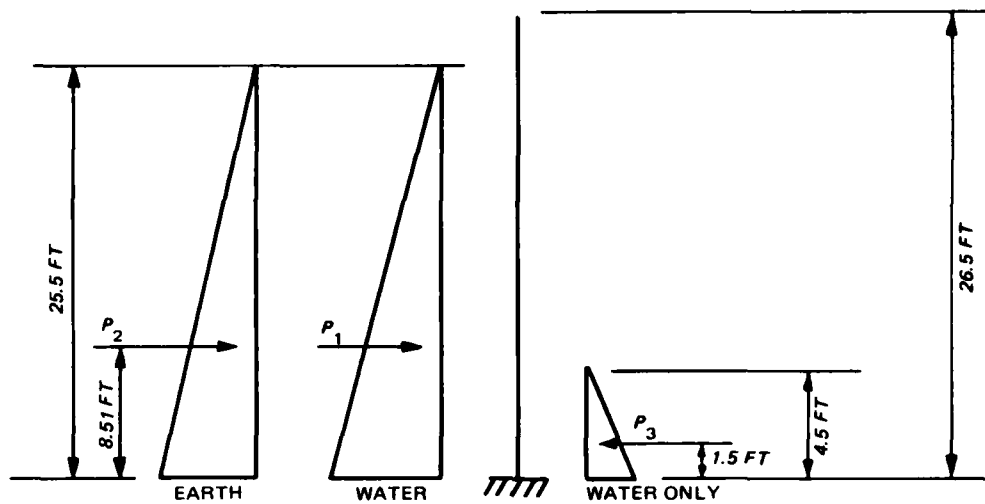


Figure A2. Loading diagram for item design

4. The effective depth  $d$  required for a given moment can be calculated by

$$d = \sqrt{\frac{M}{Rb}}$$

for  $f'_c = 3,000$  psi ,  $f_c = 0.35 f'_c = 1,050$  psi , and  $f_s = 20,000$  psi

$$R = 152^*$$

Thus

$$d = \sqrt{\frac{231,200 \times 12}{152 \times 12}}$$

$$= 39 \text{ in.}$$

---

\* Obtained from Ferguson 1965.

The required steel area can be computed by

$$A_s = \frac{M}{ad}$$

where  $a = 1.44$  for  $f_s = 20,000$  psi

Therefore

$$A_s = \frac{231.2}{1.44 \times 39}$$

$$= 4.11 \text{ sq in.}$$

Use No. 11 at 4-1/2 in. ( $A_s = 4.16$  sq in.).

Check shear at  $d$  from top of base.

$$V = \left\{ \left[ (0.0625 + 0.33 \times 0.0655) \left( 25.5 - \frac{39}{12} \right)^2 \times \frac{1}{2} \right] - \left[ 0.0625 \left( 4.5 - \frac{39}{12} \right)^2 \times \frac{1}{2} \right] \right\}$$

$$= 20.8 \text{ kips}$$

$$v = \frac{V}{bd} = \frac{20,800}{12 \times 39} = 44.4 \text{ psi} < 1.1 \sqrt{f'_c} = 60 \text{ psi}$$

5. Strength design. The factored moment at the base of the stem is

$$\begin{aligned} M_u &= 1.9M \\ &= 1.9 \times 231.2 \\ &= 439.3 \text{ kip-ft} \end{aligned}$$

The effective depth  $d$  can be determined from the following equation

$$M_u = \left[ \phi \rho f_y b d^2 \left( 1 - 0.59 \rho \left( \frac{f_y}{f'_c} \right) \right) \right]$$

Use  $\rho = 0.25 \rho_b$

$$\begin{aligned} &= 0.25 \left[ 0.85 \beta_1 \frac{f'_c}{f_y} \left( \frac{87,000}{87,000 + f_y} \right) \right] \\ &= 0.25 \left[ 0.85 \times 0.85 \times \frac{3}{40} \left( \frac{87,000}{87,000 + f_y} \right) \right] \\ &= 0.0093 \end{aligned}$$

Thus

$$439.3 \times 12 = 0.9 \times 0.0093 \times 40 \times 12 \times d^2 \left( 1 - 0.59 \times 0.0093 \times \frac{40}{3} \right)$$

$$d = 37.6 \text{ in.}$$

Use  $d = 38 \text{ in.}$

$$A_s = \rho b d$$

$$= 0.0093 \times 12 \times 38$$

$$= 4.24 \text{ sq in.}$$

Use No. 11 at 4-1/2 in. ( $A_s = 4.16 \text{ sq in.}$ )

Check shear at  $d$  from the base.

$$V_u = \left\{ \left[ 1.9(0.0625 + 0.33 \times 0.0655) \times \left( 25.5 - \frac{38}{12} \right)^2 \times \frac{1}{2} \right] - \left[ 0.0625 \left( 4.5 - \frac{38}{12} \right)^2 \times \frac{1}{2} \right] \right\}$$

$$= 39.8 \text{ kips}$$

According to Equation (11-3) of the ACI 318-77

$$V_c = 2\sqrt{f'_c} \text{ } bd$$

$$= 2\sqrt{3000} \times 12 \times 38$$

$$= 50 \text{ kips}$$

$$\phi V_c = 0.85 \times 50$$

$$42.5 \text{ kips} > V_u$$

Therefore, shear reinforcement is not required.

#### Design of heel

6. Working-stress design. Referring to Figure A3, the moment at the face of the support\* can be computed as

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\* Under most loadings, the critical heel moment is at the center line of the stem steel.

$$\begin{aligned}
 M &= (29.4 + 28.1 \times \frac{16.55}{2} + 0.525 \times \frac{(16.55)^2}{2}) \\
 &\quad - \frac{1}{2} (0.98 + 1.81)(16.55) \times \frac{16.55(2 \times 1.81 + 0.98)}{3(1.81 + 0.98)} \\
 &\quad - \frac{1}{2} (2.84)(15.72) \times \frac{1}{3} \times 15.72 \\
 &= 475.8 + 71.9 - 210.0 - 117.0 \\
 &= 220.7 \text{ kip-ft}
 \end{aligned}$$

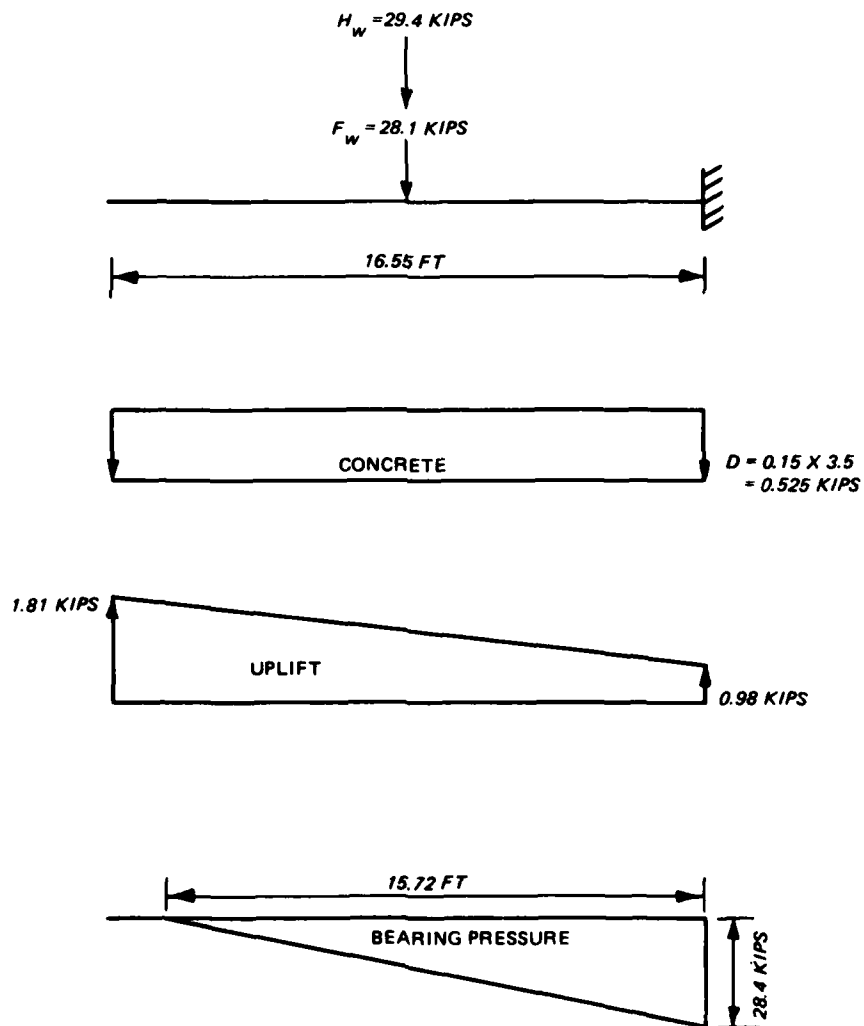


Figure A3. Loading diagram for heel design



The  $d$  required

$$d = \sqrt{\frac{M}{Rb}}$$

$$d = \sqrt{\frac{220,700 \times 12}{152 \times 12}}$$

$$d = 38.1 \text{ in.}$$

Use  $d = 38 \text{ in.}$

7. Using 4-in. concrete cover, the total thickness of the heel is

$$T = 38 + 4.5 = 42.5 \text{ in.} = 3.54 \text{ ft}$$

This is satisfactorily close to the assumed thickness of 3.5 ft used in the calculation of concrete weight.

$$\begin{aligned} A_s &= \frac{M}{ad} \\ &= \frac{220.7}{1.44 \times 38} \\ &= 4.03 \text{ sq in.} \end{aligned}$$

Use No. 11 at 4-1/2 in. ( $A_s = 4.16 \text{ sq in.}$ )

Check shear at the face of the stem.

$$\begin{aligned} V &= (29.4 + 28.1) + (0.525 \times 16.55) \\ &\quad - \frac{1}{2} (0.98 + 1.81)(16.55) \\ &\quad - \frac{1}{2} (2.84)(15.72) \\ &= 20.8 \text{ kips} \end{aligned}$$

$$v = \frac{V}{bd} = \frac{20,800}{12 \times 38} = 45.6 \text{ psi} < 60 \text{ psi}$$

8. Strength design.\*

$$\begin{aligned} M_u &= 1.9 \times 475.8 + 1.5 \times 71.9 - 1.9 \times 210.0 - 1.9 \times 117.0 \\ &= 390.6 \text{ kip-ft} \end{aligned}$$

$$M_u = \phi \rho f_y b d^2 \left( 1 - 0.59 \rho \frac{f_y}{f'_c} \right)$$

Use  $\rho = 0.25 \rho_b = 0.0093$

$$390.6 \times 12 = 0.9 \times 0.0093 \times 40 \times 12 \times d^2 \times \left( 1 - 0.59 \times 0.0093 \times \frac{40}{3} \right)$$

$$d = 35.5 \text{ in.}$$

Use  $d = 36 \text{ in.}$

and

$$A_s = \rho b d$$

$$= 0.0093 \times 12 \times 36$$

$$= 4.02 \text{ sq in.}$$

Use No. 11 at 4-1/2 in. ( $A_s = 4.16 \text{ sq in.}$ )

Check shear at the face of the stem.

---

\* A load factor of 1.9 is used for factored base reactions.

$$V_u = 1.9(29.4 + 28.1) + 1.5(0.525 \times 16.55)$$

$$- 1.9 \times \frac{1}{2} \times (1.81 + 0.98) \times 16.55$$

$$- 1.9 \times \frac{1}{2} \times (2.84 \times 15.72)$$

$$= 36 \text{ kips}$$

$$\phi V_c = 0.85(2\sqrt{f'_c} bd)$$

$$= 0.85 \times 2 \times \sqrt{3000} \times 12 \times 36$$

$$= 40.2 \text{ kips} > V_u$$

No shear reinforcement is required.

#### Design of toe

9. Working-stress design. Referring to Figure A4, the moment at the face of the support is

$$M = \frac{1}{2} \times 3.06 \times \frac{1}{3} \times (5.75)^2 + \frac{1}{2} \times 3.80 \times \frac{2}{3} \times (5.75)^2$$

$$= 58.7 \text{ kip-ft}$$

Therefore

$$d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{58.700 \times 12}{152 \times 12}}$$

$$d = 19.7 \text{ in.}$$

Use  $d = 20 \text{ in.}$

Using 4-in. concrete cover, the total thickness required for the toe at the face of the support is

$$T = 20 + 4.5 = 24.5 \text{ in.}$$

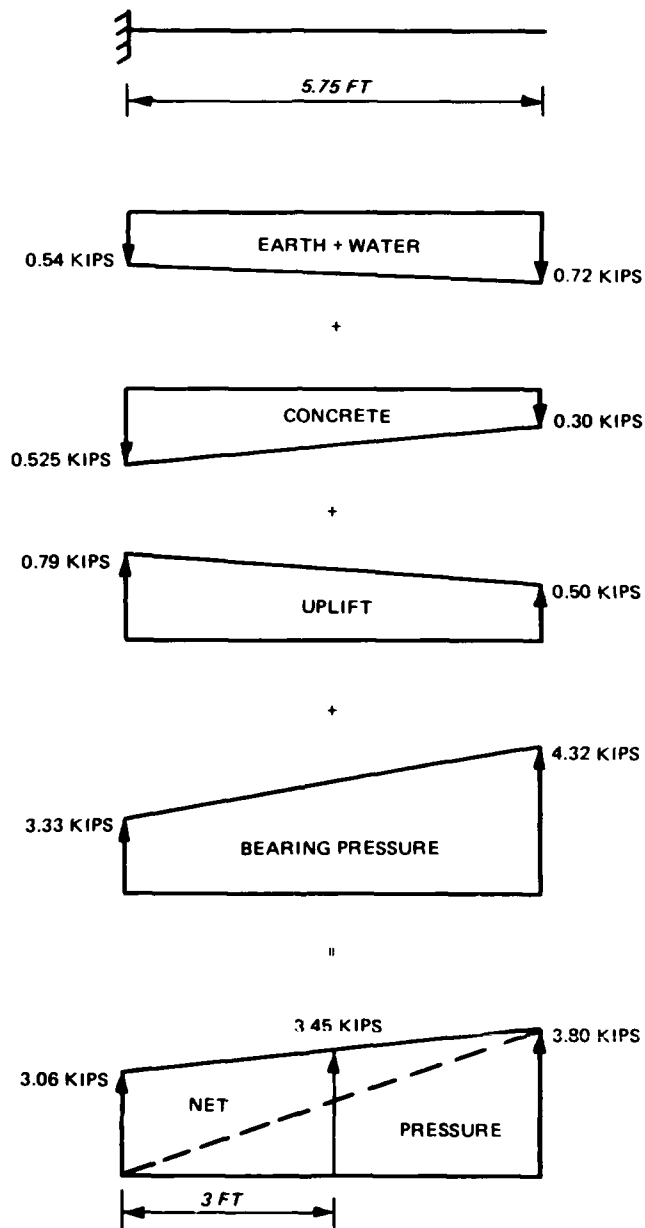


Figure A4. Loading diagram for toe design - working-stress design

10 This thickness is only about 60 percent of the assumed thickness of 42 in. used in the calculation of concrete weight. A recalculation of M and d may be necessary. However, in this design example, the recalculation is not performed.

11. The steel area required can be determined by

$$\begin{aligned} A_s &= \frac{M}{ad} \\ &= \frac{58.7}{1.44 \times 20} \\ &= 2.04 \text{ sq in.} \end{aligned}$$

Use No. 11 at 9 in. ( $A_s = 2.08 \text{ sq in.}$ )

Check shear at d from the face of the stem.

$$\begin{aligned} V &= \frac{1}{2} (3.45 + 3.80) \times (5.75 - 3.0) \\ &= 10 \text{ kips} \\ v &= \frac{V}{bd} = \frac{10,000}{12 \times 20} = 41.7 \text{ psi} < 60 \text{ psi} \end{aligned}$$

No shear reinforcement is necessary.

12. Strength design. Referring to Figure A5,\* the factored moment at the support is

$$\begin{aligned} M_u &= \frac{1}{2} \times 6.01 \times \frac{1}{3} \times (5.75)^2 + \frac{1}{2} \times 7.34 \times \frac{2}{3} \times (5.75)^2 \\ &= 114.0 \text{ kip-ft} \end{aligned}$$

$$M_u = \phi \rho f_y b d^2 \left( 1 - 0.59 \rho \frac{f_y}{f'_c} \right)$$

---

\* A load factor of 1.9 is used for factored base reaction.

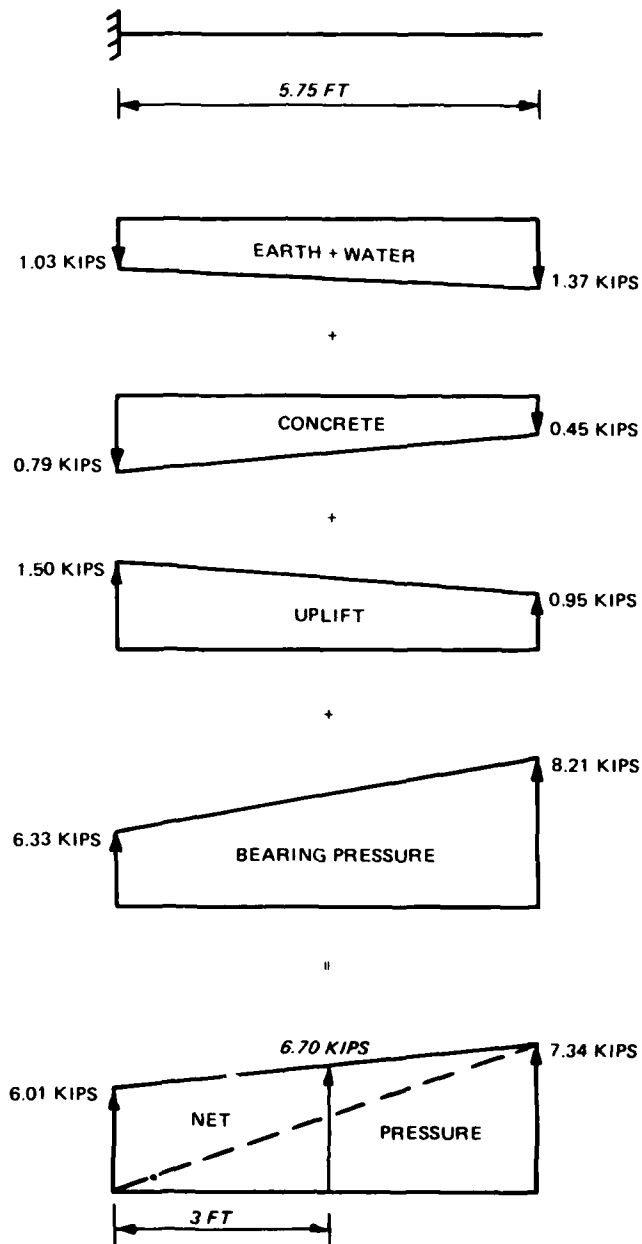


Figure A5. Loading diagram for toe design - strength design

Use  $\rho = 0.25\rho_b = 0.0093$

$$114.0 \times 12 = 0.9 \times 0.0093 \times 40 \times 12 \times d^2 \left(1 - 0.59 \times 0.0093 \times \frac{40}{3}\right)$$

$$d = 19.2 \text{ in.}$$

Use  $d = 19.5 \text{ in.}$

$$A_s = \rho b d$$

$$= 0.0093 \times 12 \times 19.5$$

$$= 2.18 \text{ sq in.}$$

Use No. 11 at 8-1/2 in. ( $A_s = 2.20 \text{ sq in.}$ )

Check shear at  $d$  from the face of the stem.

$$V_u = \frac{1}{2} (6.70 + 7.34) \times 2.75$$

$$= 19.3 \text{ kips}$$

$$\phi V_c = 0.85 \times 2 \sqrt{f'_c} b d$$

$$= 0.85 \times 2 \times \sqrt{3000} \times 12 \times 19.5$$

$$= 21.8 \text{ kips} > V_u$$

No shear reinforcement is necessary.

### Summary

13. The retaining wall design using both working-stress design and strength design methods is summarized in the following tabulation. The depths of the sections designed by the strength design method are approximately 5 percent less than those designed by the working-stress design and the steel areas required by the strength design are approximately 3 percent more than those required by the working-stress design.

|             | <u>Working-Stress<br/>Design</u> |                      | <u>Strength Design</u> |                      |
|-------------|----------------------------------|----------------------|------------------------|----------------------|
|             | <u>d</u>                         | <u>A<sub>s</sub></u> | <u>d</u>               | <u>A<sub>s</sub></u> |
| <u>Area</u> | <u>in.</u>                       | <u>sq in.</u>        | <u>in.</u>             | <u>sq in.</u>        |
| Stem        | 39.0                             | 4.11                 | 37.6                   | 4.24                 |
| Heel        | 38.1                             | 4.03                 | 35.5                   | 4.02                 |
| Toe         | 19.7                             | 2.04                 | 19.2                   | 2.18                 |

### Design of Members Subject to Combined Flexure and Axial Load

14. The following design information is given:
- Overall depth of section,  $h = 24$  in.
  - Distance from extreme compression fiber to centroid of tension reinforcement,  $d = 20$  in.
  - Width of the section,  $b = 12$  in.
  - Compressive strength of concrete,  $f'_c = 3,000$  psi
  - Yield strength of reinforcement,  $f_y = 40,000$  psi
  - Ratio of area of tension reinforcement to effective area of concrete,  $\rho = 0.00769$
  - Ratio of area of reinforcement to the gross concrete area,  $\rho_g = 0.0064$
  - Plot interaction diagrams for (1) working stress design per ACI 318-63 (ACI 1963), (2) strength design per ACI 318-77 (ACI 1977a), and (3) strength design per Part II

### Working-stress design

15. According to Equation 14-10 of ACI 318-63,



$$\begin{aligned}
P_a &= 0.34 \left( 1 + \rho_g m \right) f'_c A_g \\
&= 0.34 \left( 1 + 0.0064 \frac{40}{0.85 \times 3} \right) (3)(12)(24) \\
&= 323.3 \text{ kips}
\end{aligned}$$

16. The bending moment that could be permitted for bending alone,

$$M_f = 0.35 f'_c S_{ut}$$

where

$S_{ut}$  = the section modulus of the transformed, uncracked section

$$\begin{aligned}
&= \frac{2}{h} \left[ \frac{bh^3}{12} + (2n - 1) A_s \left( \frac{h}{2} - d_s \right)^2 \right] \\
M_f &= 0.35(3) \left( \frac{2}{24} \right) \left[ \frac{12 \times 24^3}{12} + (2 \times 9.2 - 1) \right. \\
&\quad \left. \times 0.00769 \times 12 \times 20 \left( \frac{24}{2} - 4 \right)^2 \right] \\
&= 115.8 \text{ kip-ft}
\end{aligned}$$

17. According to Equation (14-8) of ACI 318-63

$$e_b = \frac{\rho' m (d - d') + 0.1d}{(\rho' - \rho)m + 0.6}$$

For this design example,  $\rho' = 0$ , and therefore

$$\begin{aligned}
e_b &= \frac{0.1d}{0.6 - \rho m} \\
&= \frac{0.1 \times 20}{\frac{0.6 - 0.00769 \times 40}{(0.85 \times 3)}} \\
&= 4.17 \text{ in.}
\end{aligned}$$

The maximum allowable axial load  $P_{\max}$  can be calculated according to Section 1403 of the ACI 318-63

$$\begin{aligned} P_{\max} &= 0.85A_g(0.25f'_c + f_s\rho_g) \\ &= 0.85 \times 12 \times 24(0.25 \times 3 + 0.4 \times 40 \times 0.0064) \\ &= 208.7 \text{ kips} \end{aligned}$$

18. The allowable bending moment without axial load can be calculated according to Equation (14-13) of the ACI 318-63:

$$\begin{aligned} M_o &= 0.40A_s f_y j d \\ &= 0.40 \times 0.00769 \times 12 \times 20 \times 40 \times 0.891 \times 10 \\ &= 526.2 \text{ kip-in.} \\ &= 43.9 \text{ kip-ft} \end{aligned}$$

Based on the above calculation, the interaction diagram can be plotted in Figure A6.

#### Strength design, ACI 318-77

19.  $\phi P_u = 0$ . The design moment strength without axial load  $\phi M_o$  can be determined by the following equation:

$$\begin{aligned} \phi M_o &= \phi \rho f_y b d^2 \left( 1 - 0.59 \rho \frac{f_y}{f'_c} \right) \\ &= 0.9 \times 0.00769 \times 40 \times 12 \times 10^3 \\ &\quad \times \left( 1 - 0.59 \times 0.00769 \times \frac{40}{3} \right) \quad (14-13) \\ &= 1248.4 \text{ kip-in.} \\ &= 104 \text{ kip-ft} \end{aligned}$$

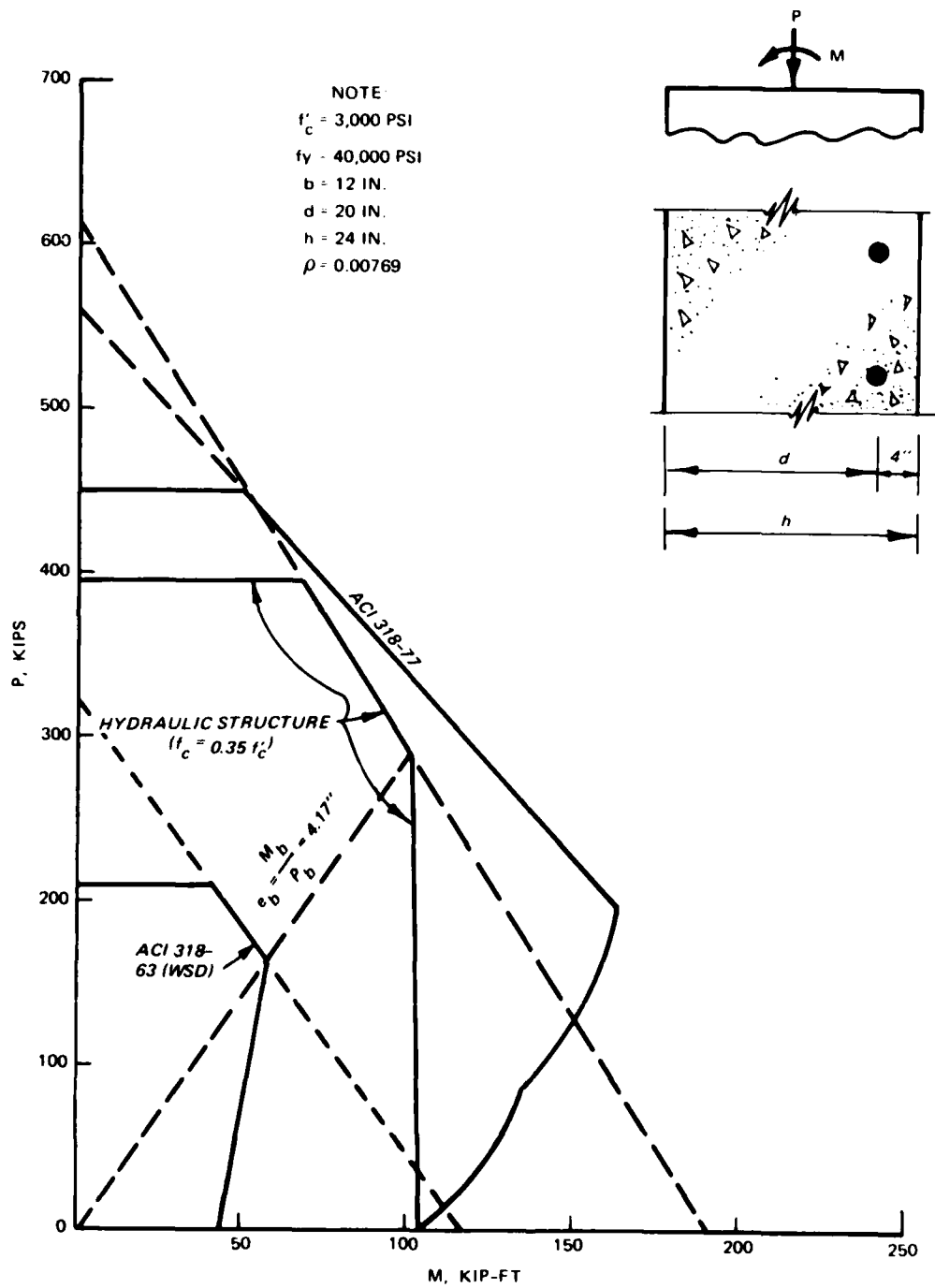


Figure A6. Interaction diagram

20.  $\phi P_u = 0.10 f'_c A_g = 86.4$  kips. Referring to Figure A7, the depth of equivalent rectangular stress block  $a$  can be determined from the following equation:

$$\frac{P_u}{\phi} = 0.85 f'_c ab - A_s f_y$$

$$\frac{86.4}{0.7} = 0.85 \times 3 \times a \times 12 - 0.00769 \times 12 \times 20 \times 40$$

$$a = 6.45 \text{ in.}$$

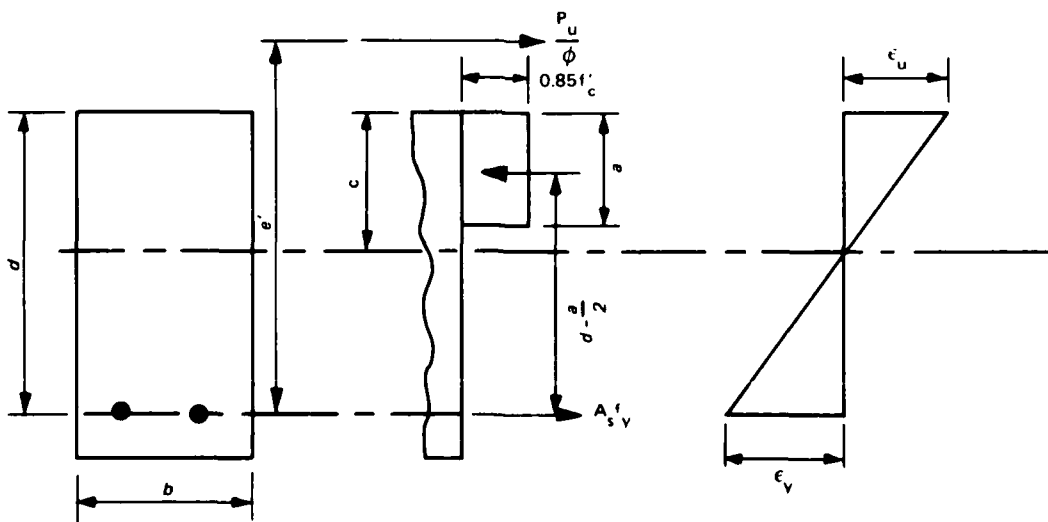


Figure A7. Stress-strain distribution for member subject to combined flexure and axial load

21. The eccentricity measured from the centroid of tension reinforcement  $e'$  can be determined from the following equation (see Figure A7).

$$\frac{F_u e'}{\phi} = 0.85 f'_c a b \left( d - \frac{a}{2} \right)$$

$$\frac{86.4 e'}{0.7} = 0.85 \times 3 \times 6.45 \times 11 \left( 20 - \frac{0.47}{2} \right)$$

$$e' = 26.8 \text{ in.}$$

The value for  $\phi M_u$  can be determined from the following:

$$e' = \frac{\phi M_u}{\phi F_u} + \left( \frac{d - d_c}{2} \right)$$

$$26.8 = \frac{\phi M_u}{86.4} + \left( \frac{20 - 4}{2} \right)$$

$$\phi M_u = 125.4 \text{ kip-ft}$$

22. Balanced condition. The balanced load strength  $\phi F_b$  and balanced moment strength  $\phi M_b$  can be computed by\*

$$\phi F_b = \phi \left( 0.85 f'_c b a_b - A_s f_y \right)$$

and

$$\phi M_b = \phi \left[ 0.85 f'_c b a_b \left( \frac{t}{2} - \frac{a_b}{2} \right) + A_s f_y \left( \frac{t}{2} - \frac{d_c}{2} \right) \right]$$

where

$$a_b = \left( \frac{87,000}{87,000 + f_y} \right) \beta_1 d$$

Substituting the given design information

---

\* Obtained from ACI (1977b).

$$a_b = \left( \frac{87,000}{87,000 + 40,000} \right) \times 0.85 \times 20$$

$$= 11.65 \text{ in.}$$

$$\phi P_b = 0.7(0.85 \times 3 \times 12 \times 11.65 - 0.00769 \times 12 \times 20 \times 40)$$

$$= 197.9 \text{ kips}$$

$$\phi M_b = 0.7 \left[ 0.85 \times 3 \times 12 \times 11.65 \left( \frac{24}{2} - \frac{11.65}{2} \right) \right.$$

$$\left. + 0.00769 \times 12 \times 20 \times 40 \left( \frac{24}{2} - 4 \right) \right]$$

$$= 1954.3 \text{ kip-in.}$$

$$= 162.9 \text{ kip-ft}$$

22.  $\phi M_u = 0$ . According to Equation (10-2) of ACI 318-77

$$\phi P_{n(\max)} = 0.90 \left[ 0.85 f'_c (A_g - A_{st}) + f_y A_{st} \right]$$

$$= 0.90 \times 0.7 \left[ 0.85 \times 3(12 \times 24 - 0.00769 \times 12 \times 20) \right.$$

$$\left. + 0.00769 \times 12 \times 20 \times 40 \right]$$

$$= 450 \text{ kips}$$

$$\phi P_u = \frac{450}{0.9} = 500 \text{ kips}$$

The interaction diagram for strength design according to ACI 318-77 is plotted in Figure A-1.

Strength design, hydraulic structures,  $f_c = 0.35f'_c$

24. According to Equation 7

$$\begin{aligned} P_A &= 0.65(1 + \rho_g m) f'_c A_g \\ &= 0.65 \left( 1 + 0.0064 \times \frac{40}{0.85 \times 3} \right) \times 3 \times 12 \times 24 \\ &= 618.0 \text{ kips} \end{aligned}$$

and

$$\begin{aligned} M_F &= 0.11 f'_c b h^2 \\ &= 0.11 \times 3 \times 12 \times 24^2 \\ &= 2281 \text{ kip-in.} \\ &= 190.1 \text{ kip-ft} \end{aligned}$$

25. The eccentricity at balanced condition  $e_b$  can be determined from Equation 6

$$\begin{aligned} e_b &= \frac{\rho' m (d - d') + 0.1d}{(\rho' - \rho) m + 0.6} \\ &= \frac{0.1d}{0.6 - \rho m} \quad (6 \text{ bis}) \\ &= 4.17 \text{ in.} \end{aligned}$$

26. The  $\phi P_b$  and  $\phi M_b$  can be determined from  $e_b$  and Equation 7:

$$\frac{\phi P_n}{P_A} + \frac{\phi M_n}{M_F} \leq 1 \quad (7 \text{ bis})$$

Substituting

$$\phi P_n = \phi P_b$$

$$\phi M_n = \phi P_b (e_b)$$

$$= 4.17 \phi P_b$$

$$P_A = 618 \text{ kips}$$

$$M_F = 190.1 \times 12$$

$$= 2281.2 \text{ kip-in.}$$

into Equation 7

$$\phi P_b = 290.2 \text{ kips}$$

and

$$\phi M_b = 290.2 \times 4.17 \div 12$$

$$= 100.8 \text{ kip-ft}$$

The maximum design axial load strength  $\phi P_n$  should be computed in accordance with Equation 5.

$$\phi P_{n(\max)} = 0.70 \phi A_g (0.85 f'_c + f_y \rho_g)$$

(5 bis)

$$= 396.0 \text{ kips}$$

The design moment strength for bending alone  $\phi M_o$  can be computed by Equation 8



$$\phi M_o = \phi \rho f_y b d^2 \left( 1 - 0.59 \rho \frac{f_y}{f'_c} \right)$$

(8, bis)

$$= 104 \text{ kip-ft}$$

27. Based on the above data, the interaction diagram for strength design of hydraulic structures is also plotted in Figure A6.

Strength design, hydraulic structures,  $f'_c = 0.45 f'_c$

28. It can be shown that  $P_A$ ,  $e_b$ ,  $\phi P_{n(\max)}$ , and  $\phi M_o$  are the same as those calculated in paragraphs 24-27.

$$M_F = 0.14 f'_c b h^2$$

$$= 0.14 \times 3 \times 12 \times 24^2$$

$$= 241.9 \text{ kip-ft}$$

and  $\phi P_b$  can be calculated by

$$\frac{\phi P_b}{P_A} + \frac{\phi P_b e}{M_F} = 1$$

Substituting into the above equation

$$P_A = 618 \text{ kips}$$

$$e = 4.17 \text{ in.}$$

$$M_F = 241.9 \times 12$$

$$= 2902.8 \text{ kip-in.}$$

then

$$\phi P_b = 327.4 \text{ kips}$$

and therefore

$$\begin{aligned}\phi M_b &= \phi P_b e \\ &= 113.8 \text{ kip-ft}\end{aligned}$$

29. The interaction diagram for hydraulic structures with  $f_c = 0.45f'_c$  is shown in Figure A8. The interaction diagrams for ACI 318-63 (WSD) (ACI 1963) and ACI 318-77 (ACI 1977a) are also plotted in Figure A8 for comparison.

#### Shear Design

##### Circular conduits

30. The stress analysis of a typical circular conduit (Figure A9) indicates that the maximum shear force and thrust at a section 45 deg from the crown (where moment is zero) are 81.25 kips and 162.5 kips, respectively, under a service load condition. Determine whether shear reinforcement is required. A 4000-psi concrete is to be used. The thickness of the conduit as shown in Figure A6 is 48 in. and has a 4-in. concrete cover.

31. Working-stress design. According to Equation (14) of EM 1110-2-2902 (OCE 1969),

$$f_t = \frac{f_c}{2} - \sqrt{\left(\frac{f_c}{2}\right)^2 + v^2} \leq 2\sqrt{f'_c}$$

where

- $f_t$  = principal tensile stress
- $f_c$  = average compressive stress
- $v$  = average shear stress

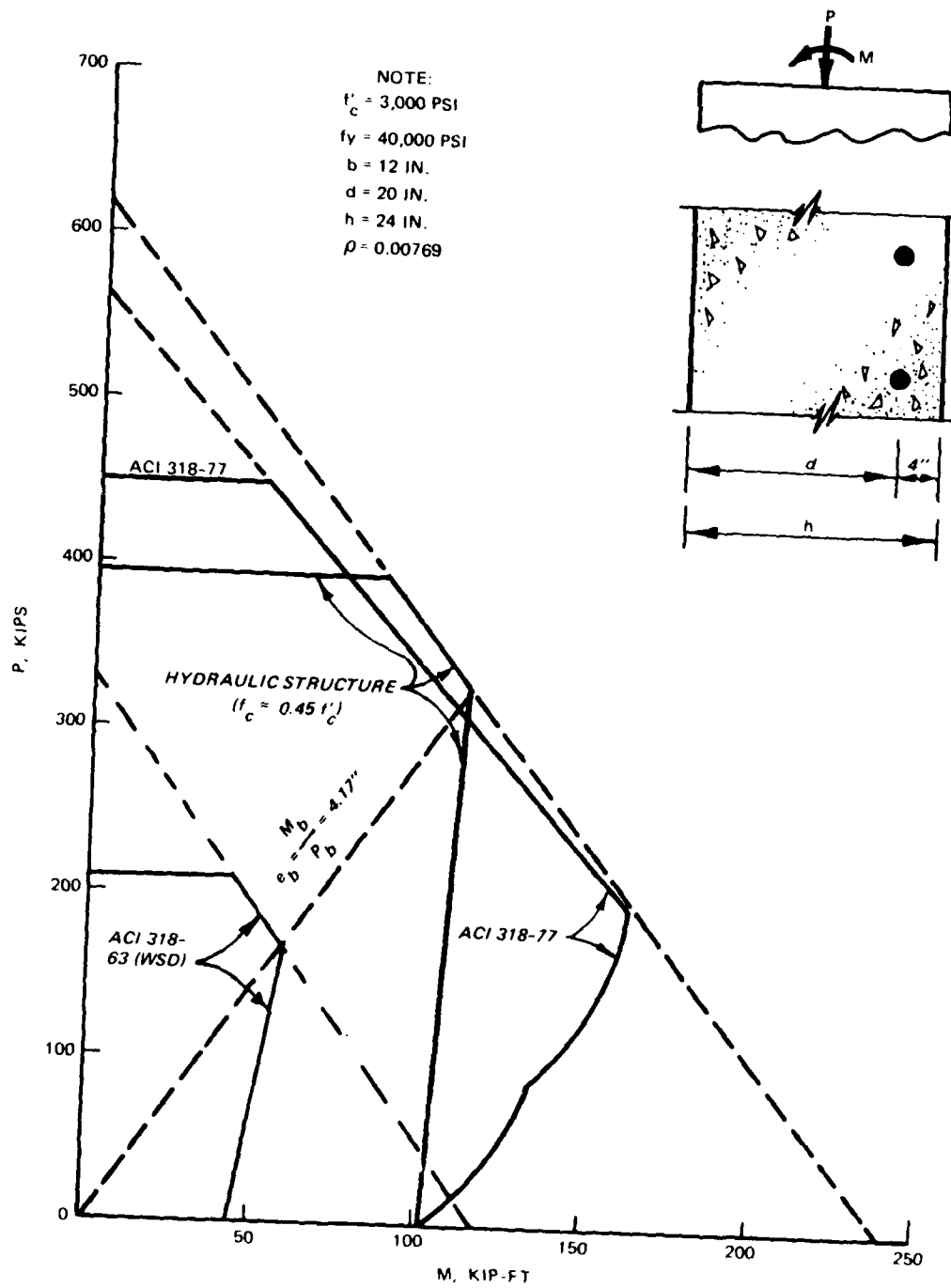


Figure A8. Interaction diagram for hydraulic structures  
 with  $f'_c = 0.45 f'_c$



$$f_c = \frac{162,500}{48 \times 12} = 282.1 \text{ psi}$$

$$v = \frac{81,250}{43.5 \times 12} = 155.7 \text{ psi}$$

$$f_t = \frac{282.1}{2} - \sqrt{\left(\frac{282.1}{2}\right)^2 + (155.7)^2}$$

$$= 69 \text{ psi} < 2\sqrt{f'_c} = 126.5 \text{ psi}$$

Therefore, no shear reinforcement is required.

32. Strength design. Since  $R/d = 11 \times 12/43.5 = 3.03 > 2.25$ , the nominal shear strength provided by concrete,  $V_c$ , shall be computed by Equation 12:

$$V_c = 4\sqrt{f'_c} \left( \sqrt{1 + \frac{\frac{N_u}{A_g}}{4\sqrt{f'_c}}} \right) bd \quad (12 \text{ bis})$$

where

$$N_u = 162.5 \times 1.9 = 308.8 \text{ kips}$$

$$A_g = 12 \times 48 = 576 \text{ sq in.}$$

$$bd = 12 \times 43.5 = 522 \text{ sq in.}$$

$$f'_c = 4000 \text{ psi}$$

Thus,

$$V_c = 4\sqrt{4000} \left( \sqrt{1 + \frac{\frac{308,800}{576}}{4\sqrt{4000}}} \right) \times 522$$

$$= 233,227 \text{ lb}$$

$$= 233.2 \text{ kips}$$

$$\phi V_c = 0.85 \times 233.2$$

$$= 198.2 \text{ kips}$$

$$V_u = 1.9 \times 81.25$$

$$= 154.4 \text{ kips}$$

Since  $V_u < \phi V_c$ , shear reinforcement is not required.

#### Box culverts or conduits

33. For a typical rectangular one-cell reinforced concrete box culvert (Figure A10), the following design information for the horizontal member is given:

- a. Clear span,  $l_n = 10 \text{ ft}$
- b. Span between points of contraflexure,  $l' = 9 \text{ ft}$
- c. Uniform service load,  $w = 7.2 \text{ kips/ft}$
- d. Thrust,  $N = 14.8 \text{ kips}$
- e. Effective depth,  $d = 24 \text{ in.}$
- f. Overall thickness,  $28 \text{ in.}$
- g. Steel ratio,  $\rho = 0.005$
- h. Concrete compressive strength,  $f'_c = 4000 \text{ psi}$

Design shear reinforcement for the horizontal member.

34. Working-stress design. The unit shearing stress of box culvert can be determined using Equation (15) of FM 1110-2-2902 (OCE 1969).

$$v_{pc} = 11,000 \frac{(0.046 + \rho) \left( 19 + \frac{N}{V} \right)}{\left( 19 + \frac{l'}{d} \right)} \sqrt{\frac{f'_c}{4000}}$$

where

$V$  = total shear at point of contraflexure

$$= \frac{wl'}{2}$$

$$= \frac{7.2 \times 9}{2} = 32.4 \text{ kips}$$

$$N/V = 14.8/32.4 = 0.46$$

$$l'/d = 9 \times 12/24 = 4.5$$

$$\rho = 0.005$$

$$f'_c = 4000 \text{ psi}$$

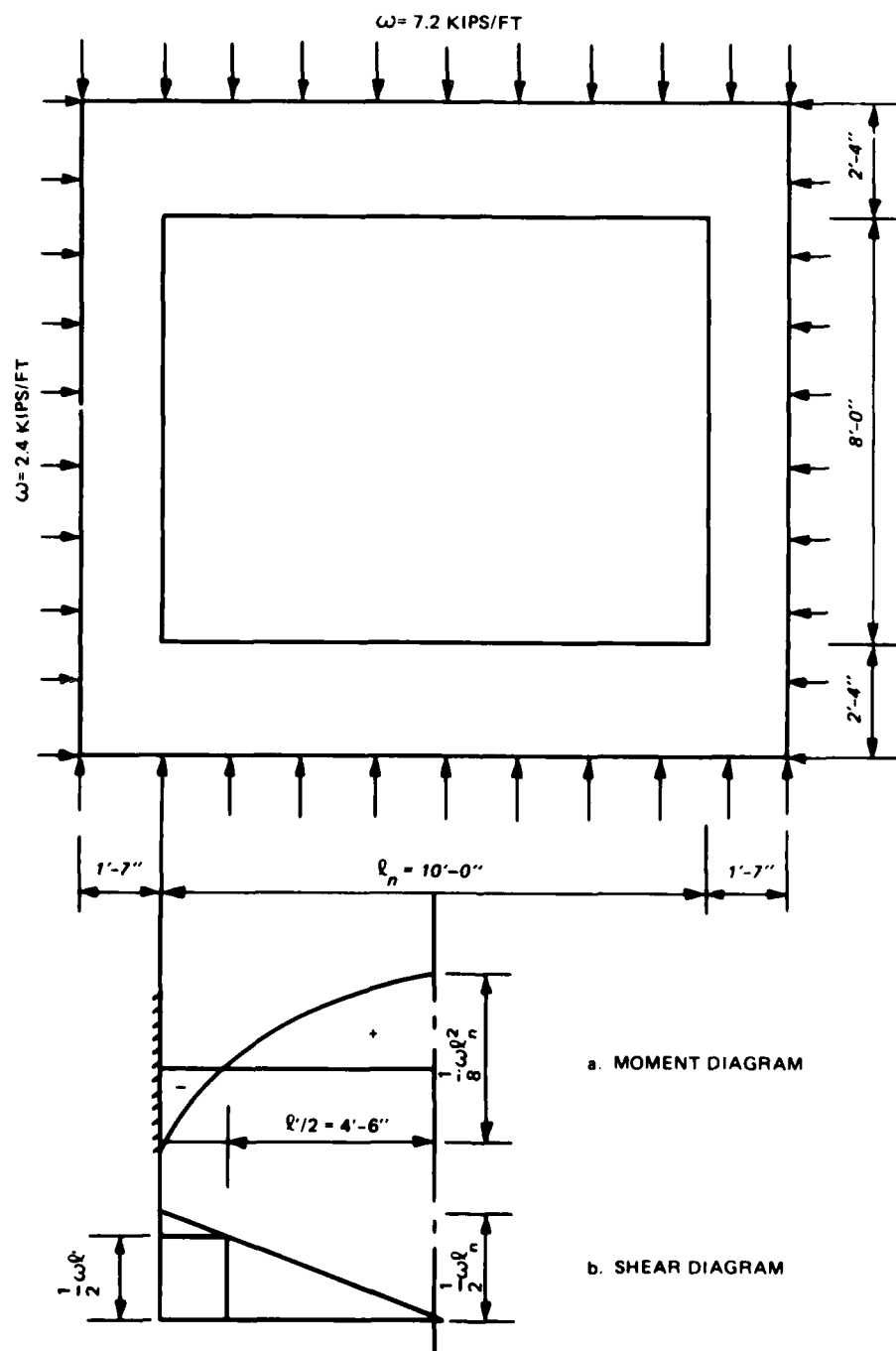


Figure A10. Rectangular one-cell reinforced concrete box culvert section

Therefore,

$$v_{pc} = 11,000 \frac{(0.046 + 0.005)(12 + 0.46)}{(19 + 4.5)} \sqrt{\frac{4000}{4000}}$$

$$= 297.4 \text{ psi}$$

$$v = \frac{V}{bd} = \frac{32,400}{12 \times 24} = 112.5 \text{ psi}$$

The factor of safety in shear for  $3 < l'/d < 5$  is

$$FS = 1.25 + \frac{l'}{4d}$$

$$= 1.25 + \frac{9 \times 12}{4 \times 24}$$

$$= 2.38$$

$$\frac{v_{pc}}{FS} = \frac{297.4}{2.38} = 125.2 \text{ psi} > v = 112.5 \text{ psi}$$

Shear reinforcement is not required.

35. Strength design. Since  $l/d = 10/2 = 5 < 6$ , Equation 10 is applicable, and shear should be checked at  $0.15l_n$  from the face of the support:



$$\begin{aligned}
V_c &= \left[ \left( 11.5 - \frac{\ell_n}{d} \right) \sqrt{f'_c} \sqrt{1 + \frac{\frac{N_u}{A}}{5\sqrt{f'_c}}} \right] bd \\
&= (11.5 - 5) \sqrt{4000} \sqrt{1 + \frac{83.7}{5\sqrt{4000}}} \times 12 \times 24 \\
&= 132.6 \text{ kips} < 2.1 \left( 11.5 - \frac{\ell_n}{d} \right) \sqrt{f'_c} bd = 248.6 \text{ kips} \quad (10 \text{ bis}) \\
\phi V_c &= 0.85 \times 132.6
\end{aligned}$$

$$= 112.7 \text{ kips}$$

$V_u$  at  $0.15\ell_n$  from the face of the support is

$$\begin{aligned}
V_u &= w \left( \frac{\ell_n}{2} - 0.15\ell_n \right) \\
&= 1.9 \times 7.2 \left( \frac{10}{2} - 0.15 \times 10 \right) \\
&= 47.9 \text{ kips} < \phi V_c
\end{aligned}$$

Shear reinforcement is also not required at  $0.15\ell_n$  from the face of the support.

#### Horseshoe-shaped conduits

36. The elastic analysis of a horseshoe-shaped conduit (Figure A11) shows that the maximum shear occurs at Section 10. The moment, shear, and thrust at this section are 37 kips-ft, 19.7 kips, and 13 kips, respectively. The thickness of the Section 10 is 2.1<sup>2</sup> ft and the effective depth is 1.94 ft. Concrete having  $f'_c = 4000$  psi is to be used.

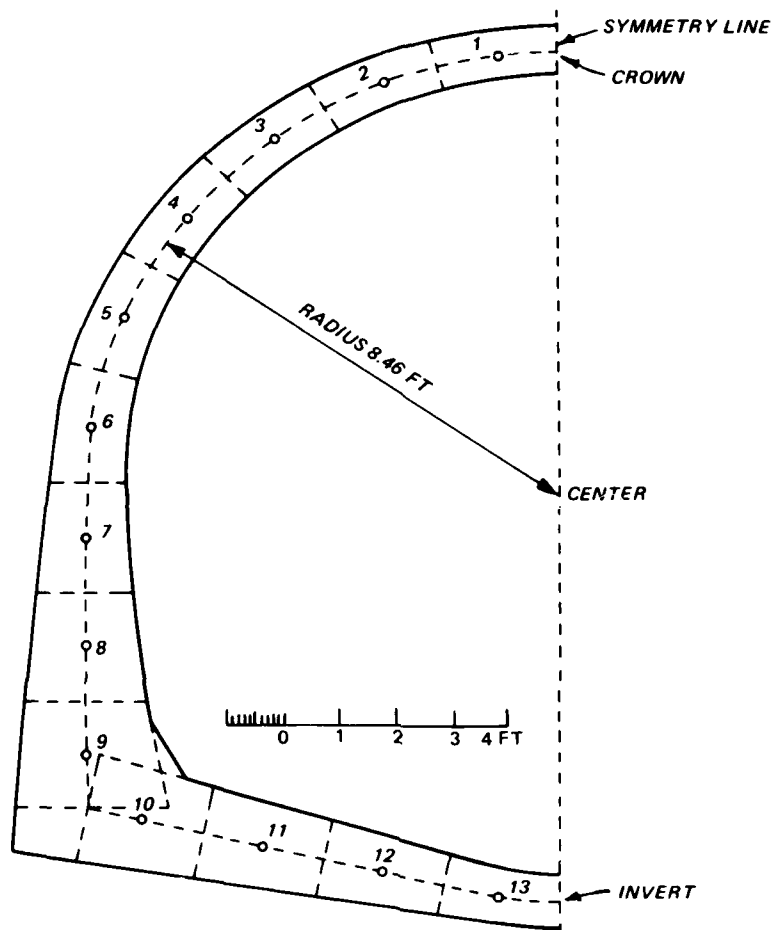


Figure A11. Horseshoe-shaped conduit

The steel ratio  $\rho$  is 0.01 . Determine whether shear reinforcement is required.

37. Working-stress design. According to Equation (B-3) of the ACI 318-77,

$$v_c = \sqrt{f'_c} + 1300\rho \frac{Vd}{M}$$

where

$$V = 19.7 \text{ kips}$$

$$\frac{Vd}{M} = \frac{19.7 \times 1.94 \times 12}{37 \times 12}$$

$$= 1.03$$

$$\rho = 0.01$$

$$f'_c = 4000 \text{ psi}$$

$$v_c = \sqrt{4000} + 1300 \times 0.01 \times 1.03$$

$$= 76.6 \text{ psi}$$

$$v = \frac{19,700}{1.94 \times 12 \times 12} = 70.5 \text{ psi} < v_c$$

No shear reinforcement is required.

38. Strength design. Since the maximum shear occurs at Section 10 where moment and thrust also exist, the provision of Section 11.8.2.2 of ACI 318-77 applies:

$$v_c = \left( 1.9 \sqrt{f'_c} + 2500 \rho \frac{V_u d}{M_m} \right) bd$$

and

$$M_m = M_u - N_u \left( \frac{4h - d}{8} \right)$$

From the given information

$$M_u = 37 \times 1.9 = 70.3 \text{ kip-ft}$$

$$N_u = 13 \times 1.9 = 24.7 \text{ kips}$$

$$V_u = 19.7 \times 1.9 = 37.4 \text{ kips}$$

and

$$M_m = 70.3 - 34.7 \left( \frac{4 \times 5.18 - 1.94}{8} \right)$$

$$= 49.4 \text{ kip-ft}$$

$$V_c = \left( 1.9\sqrt{4000} + 2500 \times 0.01 \frac{37.4 \times 1.94}{49.4} \right) \times 12 \times 1.94 \times 12$$

$$= 43.8 \text{ kips}$$

$$\phi V_c = 0.85 \times 43.8$$

$$= 37.2 \text{ kips}$$

Since this value is almost equal to  $V_u$  (37.4 kips), the shear reinforcement is not considered necessary.

#### Intake structure wall

39. For an intake structure wall (Figure A12), the following information is given:

- a. Clear span,  $l_n = 14 \text{ ft}$
- b. Overall thickness,  $h = 36 \text{ in.}$
- c. Effective depth,  $d = 33.5 \text{ in.} = 2.79 \text{ ft}$
- d. Moment at face of support,  $M_s = 349 \text{ kip-ft}$
- e. Thrust,  $N = 116 \text{ kips}$
- f. Shear at face of support,  $V_s = 83 \text{ kips}$
- g. Uniform load at compression face,  $w = 11.8 \text{ kips/ft}$
- h. Concrete compressive strength,  $f'_c = 3000 \text{ psi}$
- i. Reinforcement ratio,  $\rho = 0.006$

Determine if shear reinforcement is required.

#### 40. Working-stress method.

- a. Check shear at  $d$  from the face of the support.

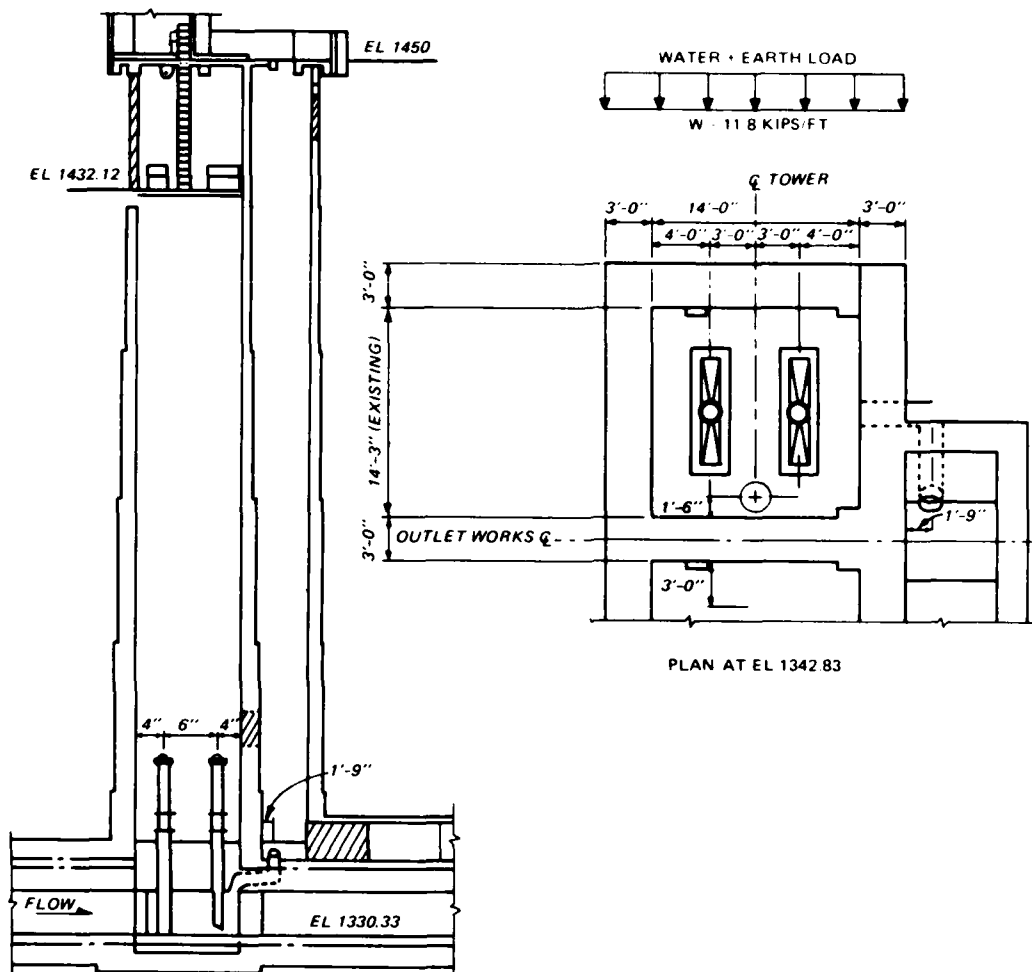


Figure A12. Intake structure wall

$$V_d = V_s - wd$$

$$= 83 - 11.8 \times 2.79$$

$$= 50.1 \text{ kips}$$

For members subject to axial compression,  $v_c$  may be computed according to Equation B-4 of ACI 318-77.

$$v_c = 1.1 \left( 1 + 0.006 \frac{N}{A_g} \right) \sqrt{f'_c}$$

where

$$\frac{N}{A_g} = \frac{116,000}{12 \times 36} = 268.5 \text{ psi}$$

$$v_c = 1.1(1 + 0.006 \times 268.5) \sqrt{3000}$$

$$= 157.3 \text{ psi}$$

$$v = \frac{V_d}{bd} = \frac{50,100}{12 \times 33.5} = 124.6 \text{ psi} < v_c = 157.3 \text{ psi}$$

Shear reinforcement is not required.

- b. Check shear at points of contraflexure. Since the moment at any point  $M_x$  along the wall can be computed by

$$M_x = M_s + \frac{wx^2}{2} - V_s x$$

The point of contraflexure can be determined by letting  $M_x = 0$  and solve for  $x$ .

$$0 = 249 + \frac{11.8(x^2)}{2} - 83x$$

$$x = 4.3 \text{ ft}$$

The span between points of contraflexure  $l'$  can be calculated by

$$l' = l_n - 2x$$

$$= 14 - 2 \times 4.3$$

$$= 5.4 \text{ ft}$$

The shear force at point of contraflexure  $V$  is

$$V = \frac{w\ell'}{2}$$

$$= \frac{11.8 \times 5.4}{2} = 31.9 \text{ kips}$$

and

$$v = \frac{V}{bd} = \frac{31,900}{12 \times 33.5}$$

$$= 79.4 \text{ psi}$$

41. Since the structural conditions of this intake structure wall are similar to the box culverts, Equation (15) of EM 1110-2-0902 (OCE 1969) shall apply

$$v_{pc} = 11,000 \frac{(0.046 + \rho) \left(12 + \frac{N}{V}\right)}{\left(19 + \frac{\ell'}{d}\right)} \sqrt{\frac{f'_c}{4000}}$$

where

$$N/V = \frac{116}{31.9} = 3.6$$

$$\ell'/d = \frac{5.4}{2.79} = 1.94$$

$$\rho = 0.006$$

$$f'_c = 3000$$

$$v_{pc} = 11,000 \frac{(0.046 + 0.006)(12 + 3.6)}{(19 + 1.94)} \sqrt{\frac{3000}{4000}}$$

$$= 369 \text{ psi}$$

The factor of safety in shear for  $l'/d < 3$  is 2.0 according to EM 1110-2-2902 (OCE 1969).

$$\frac{v_{pc}}{FS} = \frac{369}{2} = 184.5 \text{ psi} > 79.4 \text{ psi}$$

42. Strength design.

- a. Check shear at  $d$  from the face of the support. Since the intake structure wall is subjected to moment, shear, and axial compression, the provisions of Section 11.3.2.2 of ACI 318-77 shall apply.

$$\begin{aligned} (M_u)_d &= 1.9 \left( M_s + \frac{\omega d^2}{2} - V_s d \right) \\ &= 1.9 \left( 249 + \frac{11.8 \times 2.79 \times 2.79}{2} - 83 \times 2.79 \right) \\ &= 120.4 \text{ kip-ft} \end{aligned}$$

$$\begin{aligned} (V_u)_d &= 1.9(V_s - \omega d) \\ &= 1.9(83 - 11.8 \times 2.79) \\ &= 95 \text{ kips} \end{aligned}$$

$$N_u = 1.9 \times 116 = 220.4 \text{ kips}$$

According to Equation 11-7 of ACI 318-77

$$\begin{aligned} M_m &= M_u - N_u \left( \frac{l_h - d}{8} \right) \\ &= 120.4 - 220.4 \left( \frac{4 \times 36 - 33.5}{8} \right) \\ &= -2923.9 \text{ kip-ft} \end{aligned}$$



Since  $M_n$  is negative,  $V_c$  shall be computed by Equation (11-8) of ACI 318-77

$$\begin{aligned}
 V_c &= 3.5 \sqrt{f'_c} \, b d \sqrt{1 + \frac{N_u}{500 A_g}} \\
 &= 3.5 \sqrt{3000} \times 12 \times 33.5 \sqrt{1 + \frac{220,400}{500 \times 12^2 \times 36}} \\
 &= 109.5 \text{ kips}
 \end{aligned}$$

$$\phi V_c = 0.85 \times 109.5$$

$$= 93 \text{ kips}$$

This value is approximately 2 percent less than the  $V_u$  at Section d from the face of the support and the design is considered acceptable without shear reinforcement.

- b. Check shear at  $0.15 \ell_n$  from the face of the support.  
From the given design information

$$\frac{\ell_n}{d} = \frac{14}{2.79} = 5.0 < 6$$

Therefore, Equation 10 is applicable:

$$\begin{aligned}
 V_c &= \left[ \left( 11.5 - \frac{\ell_n}{d} \right) \sqrt{f'_c} \sqrt{1 + \frac{N_u}{5 A_g \sqrt{f'_c}}} \right] b d \quad (10, bis) \\
 &= (11.5 - 5) \sqrt{3000} \sqrt{1 + \frac{510}{5 \sqrt{3000}}} \times 12 \times 36 \\
 &= 260.2 \text{ kips} < 2.1 \left[ \left( 11.5 - \frac{\ell_n}{d} \right) \sqrt{f'_c} b d \right] = 323 \text{ kips}
 \end{aligned}$$

$$\phi V_c = 0.85 \times 260.2$$

$$= 221.2 \text{ kips}$$

The factored shear force at  $0.15\ell_n$  from the face of the support can be computed:

$$\begin{aligned} V_u &= 1.9[V_s - \omega(0.15\ell_n)] \\ &= 1.9(83 - 11.8 \times 0.15 \times 14) \\ &= 110.6 \text{ kips} \end{aligned}$$

Since  $V_u < \phi V_c$ , the shear reinforcement is not required at  $0.15\ell_n$  from the face of the support. Since  $\ell_n/d = 5$  and the intake structure wall is loaded at the compression face, the provisions for deep flexural members given in Section 11.8 of ACI 318-77 may be used. According to Equation (11-29) of ACI 318-77, the nominal shear strength,  $V_c$  at  $0.15\ell_n$  from the face of the support, can be computed by

$$V_c = \left(3.5 - 2.5 \frac{M_u}{V_u d}\right) \left(1.9 \sqrt{f'_c} + 2500\rho \frac{V_u d}{M_u}\right) bd$$

where

$$\begin{aligned} M_u &= 1.9 \left[ M_s + \frac{\omega(0.15\ell_n)^2}{2} - V_s(0.15\ell_n) \right] \\ &= 1.9 \left[ 249 + \frac{11.8(0.15 \times 14)^2}{2} - 83(0.15 \times 14) \right] \\ &= 191.4 \text{ kip-ft} \end{aligned}$$

$$V_u = 110.6 \text{ kips}$$

$$d = 2.79 \text{ ft}$$

$$f'_c = 3000 \text{ psi}$$

$$\rho = 0.006$$

$$b = 12 \text{ in.}$$

$$d = 33.5 \text{ in.}$$

and

$$3.5 - 2.5 \frac{M_u}{V_u d} = 2.0 < 2.5$$

Therefore

$$\begin{aligned} V_c &= 2.0 \left( 1.9 \sqrt{f'_c} + 2500 \rho \frac{V_u d}{M_u} \right) b d \\ &= 2.0 \left( 1.9 \sqrt{3000} + 2500 \times 0.006 \right. \\ &\quad \times \left. \frac{110.6 \times 2.79}{191.4} \right) \times 12 \times 33.5 \\ &= 103.1 \text{ kips} \end{aligned}$$

Comparison of this value with the  $V_c$  of 260.2 kips computed by Equation 7 indicates that the  $V_c$  value computed by Equation (11-29) of ACI 318-77 is too conservative because the effects of axial loads on the shear strength of concrete are not considered by the ACI equation. Therefore, the ACI equation shall not be used if the deep flexural member is subjected to significant axial compression, in addition to moment and shear.

APPENDIX B: COMPARISON OF DESIGN CRITERIA  
FOR HYDRAULIC STRUCTURES AND  
ACI 318-77 FOR BUILDINGS

Table B1

Comparison of Design Criteria for Hydraulic Structures  
and ACI 318-77 for Buildings

| Item                                    | Hydraulic Structures  | Buildings   |
|---|---|---|
| Required strength                       | $U = 1.5D + 1.9(L + H_w + H_p + F_w + F_p + F_u)$ $U = 0.9D + 1.9(H_w + H_p + F_w + F_p + F_u)$ $U = 0.75[1.5D + 1.9(L + H_w + H_p + F_w + F_p + F_u + p + w + T)]$ $U = 0.75[1.5D + 1.9(L + H_w + H_p + F_w + F_p + F_u + P + E + T)]$ | $U = 1.4D + 1.7L$ $U = 0.75 (1.4D + 1.7L + 1.7W)$ $U = 0.9D + 1.3W$ $U = 0.75 (1.4D + 1.7L + 1.9E)$ $U = 1.4D + 1.7L + 1.7H$ $U = 1.4D + 1.7L + 1.4F$ $U = 0.75 (1.4D + 1.4T + 1.7L)$ $U = 1.4 (D + T)$ |
| Design strength for reinforcement       | 40,000 psi  | 40,000 to 80,000 psi  |
| Distribution of reinforcement           | 12 in.  | $Z = 175 \text{ kips/in.} - \text{Interior exposure}$ $= 145 \text{ kips/in.} - \text{Exterior exposure}$   |
| Shrinkage and temperature reinforcement | Per EM 1110-2-2103 (OCE 1971), depending on degree of restraint   | $0.2\% \text{ for GR40 or 50}$ $0.18\% \text{ for GR60}$ $\frac{0.18\% \times 60,000}{f_y} \text{ for } f_y > 60,000$   |

(Continued)

(Sheet 1 of 3)

Table E1 (Continued)

| Item  | Hydraulic Structures  |                   | Buildings         |   |
|---|---|-------------------|-------------------|---|
|   | Deflections need not be computed if $\rho < 0.25\rho_b$   | Check deflections | Check deflections |   |
| Maximum reinforcement ratio                 | $0.25\rho_b$  |                   |                   | $0.75\rho_b$  |
| Minimum reinforcement ratio                 | 0.5 percent   |                   |                   | $\frac{200}{f_y}$   |
| Maximum axial load strength                 | $\phi F_{n(max)} = 0.70\phi A_g (0.85f'_c + f_y \rho_g)$  |                   |                   | $\phi P_{n(max)} = 0.80\phi [0.85f'_c (A_g - A_{st}) + f_y A_{st}]$ |
| Combined flexure and axial load             | $e_b = \frac{\rho'_m(d - d') + 0.1d}{(\rho' - \rho)m + 0.6}$<br>$\frac{f'_m}{f'_c} + \frac{f'_m}{f'_c} < 1$ |                   |                   | Based on stress and strain compatibility                            |
| Nominal shear strength provided by concrete | Straight members with $l_u/d > 6$   |                   |                   | None  |

$$V_c = \left[ 11,000 \frac{(0.06 + \rho) \left( 12 + \frac{V_u}{V_c} \right)}{\left( 19 + \frac{V_u}{V_c} \right)} \right] \sqrt{\frac{f'_c}{1000}} bd$$

(Continued)

(Sheet 2 of 3)

Table P1 (Concluded)

| Iter. | Hydraulic Structures  | Buildings |
|-------|---|-----------|
|       | Straight members with $2 < \ell_n/d < 6$  |           |
|       | $V_c = \left[ \left( 11.5 - \frac{\ell_n}{d} \right) \sqrt{f'_c} \sqrt{1 + \frac{\frac{N}{A} \frac{u}{g}}{5 \sqrt{f'_c}}} \right] bd$ | None      |
|       | Curved cast-in-place members with $R/d > 2.25$  |           |
|       | $V_c = \left( 4 \sqrt{f'_c} \sqrt{1 + \frac{\frac{N}{A} \frac{u}{g}}{4 \sqrt{f'_c}}} \right) bd$                                      | None      |

APPENDIX C: DERIVATION OF LOAD FACTORS AND STRENGTH  
REDUCTION FACTORS USING PROBABILISTIC APPROACH

Introduction

1. The purpose of this appendix is to show how the load factors and strength reduction factors can be determined in a rational manner, taking into account the inherent uncertainties of the resistances and load effects.

2. The "first-order" or "second-moment" probabilistic method developed by Cornell (1969), Lind (1971), Rosenblueth and Esteva (1972), and others (Ravindra, Heaney, and Lind 1969) was used. This is a simplified method that uses only two statistical parameters: mean values and the coefficient of variation of the relevant variables.

Basic Theory

3. If  $R$  and  $U$  represent the distributions of strengths and loads, respectively, any given structure will fail if  $U > R$ . Thus the probability of failure  $P_f$  is the probability that  $U > R$ , or

$$P_f = P [(R - U) < 0] \quad (C1)$$

or alternatively

$$P_f = P \left( \frac{R}{U} < 1.0 \right) \quad (C2)$$

Since  $\ln 1.0 = 0$ , Equation C2 can be expressed as

$$P_f = P \left[ \ln \left( \frac{R}{U} \right) < 0 \right] \quad (C3)$$

4. In other words, the probability of failure can be expressed as the probability that  $\ln(R/U)$  is less than zero. This probability is represented by the shaded area in Figure C1. As shown, the distance of



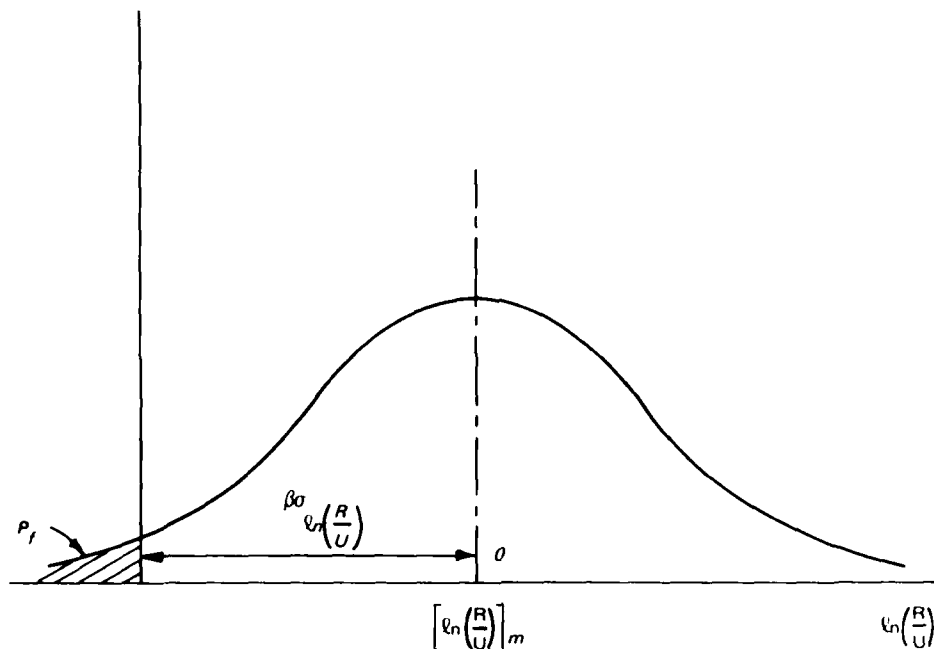


Figure C1. Definition of safety index

the mean of  $\ln(R/U)$ ,  $[\ln(R/U)]_m$ , with respect to the origin can conveniently be measured as a number  $\beta$  times the standard deviation  $\sigma$  of  $\ln(R/U)$ .

5. If the actual distribution of  $\ln(R/U)$  were known, and if a value of the probability of failure could be agreed upon, a complete probability-based set of design criteria could be established. Unfortunately, so much information is not known. The distribution shape of each of the many variables (e.g. material, load) has an influence on the shape of the distribution of  $\ln(R/U)$ . At best only the means and the standard deviations of the many variables involved in the make-up of the resistance and the load effect can be estimated. However, this information is enough to build a first-order approximate design criterion by stipulating the following design condition.

$$\left[ \ln\left(\frac{R}{U}\right) \right]_m \geq \beta \sigma \ln\left(\frac{R}{U}\right) \quad (C1)$$

6. Equation C4 can be simplified by using first-order probability theory as follows:

$$\left[ \ln\left(\frac{R}{U}\right) \right]_m \approx \ln\left(\frac{R}{U}\right)_m \approx \ln\left(\frac{R_m}{U_m}\right) \quad (C5)$$

and

$$\begin{aligned} \sigma^2 \ln\left(\frac{R}{U}\right) &\approx \left[ \frac{\partial \ln\left(\frac{R}{U}\right)}{\partial R} \right]_m^2 \sigma_R^2 + \left[ \frac{\partial \ln\left(\frac{R}{U}\right)}{\partial U} \right]_m^2 \sigma_U^2 \\ &\approx \frac{\sigma_R^2}{R_m^2} + \frac{\sigma_U^2}{U_m^2} \end{aligned} \quad (C6)$$

Since  $\sigma_R/R_m = V_R$  and  $\sigma_U/U_m = V_U$ , the coefficients of variation of the resistance  $R$  and the load effect  $U$ , Equation C4 becomes

$$\ln\left(\frac{R_m}{U_m}\right) \geq \beta \sqrt{V_R^2 + V_U^2} \quad (C7)$$

7. To separate the resistance and load terms, Lind (1971) proposed a linear approximation to the square root term in Equation C7

$$\sqrt{V_R^2 + V_U^2} \approx \alpha (V_R + V_U) \quad (C8)$$

where  $\alpha$  is a constant equal to 0.75. This approximation is good to within  $\pm 6$  percent for the range  $1/3 < V_R/V_U < 3$ . By this linearization, it is possible to write Equation C7 as

$$\ln \frac{R_m}{U_m} \geq \beta \alpha V_R + \beta \alpha V_U \quad (C9)$$

or

$$\frac{R_m}{U_m} \geq e^{(\beta\alpha V_R + \beta\alpha V_U)} \quad (C10)$$

Rearranging this gives

$$R_m \left( e^{-\beta\alpha V_R} \right) \geq U_m \left( e^{\beta\alpha V_U} \right) \quad (C11)$$

This resembles the current ACI 318-77 (ACI 1977a) format in that the average strength  $R_m$  is multiplied by a factor less than 1.0 and the average load  $U_m$  is multiplied by a factor greater than 1.0. However, when the designer uses the code design equations and the specified strengths, he computes the design strength  $R$  rather than the mean strength  $R_m$ . Similarly, the design is based on values of specified  $U$  rather than the mean load  $U_m$ . Therefore,  $\gamma_R$  and  $\gamma_U$  are defined

$$R_m = R\gamma_R \quad (C12)$$

and

$$U_m = U\gamma_U \quad (C13)$$

then Equation C11 can be rewritten as

$$R\gamma_R \left( e^{-\beta\alpha V_R} \right) \geq U\gamma_U \left( e^{\beta\alpha V_U} \right) \quad (C14)$$

or

$$R \phi \geq U \lambda \quad (C15)$$

where  $\phi$  is the strength reduction factor and  $\lambda$  is the load factor.  
Thus

$$\phi = \gamma_R e^{-\beta \alpha V_R} \quad (C16)$$

and

$$\lambda = \gamma_U e^{\beta \alpha V_U} \quad (C17)$$

Before values of  $\phi$  and  $\lambda$  can be derived, an appropriate level of safety defined by the safety index  $\beta$  must be defined, and  $\gamma_R$ ,  $V_R$ ,  $\gamma_U$ , and  $V_U$  estimated. The choice of  $\beta$  and the calculation of these terms will be given in the following sections.

#### Choice of Acceptable Probability of Failure

8. There are two ways in which guidance in selecting  $\beta$  can be obtained. Relationships given in Equation C16 and C17 can be used to calculate the value of  $\beta$  corresponding to the load factors and strength reduction factors in the current codes. If these  $\beta$  values and the related probabilities of failure are felt to be realistic, they can be used to derive new values of  $\phi$  and  $\lambda$  for use in the new criteria. If not, more appropriate target values of  $\beta$  can be selected on the basis of the performance of the current code or engineering judgment. This approach is called "calibration" since the new criteria are calibrated or made to agree with a target established by a study of the old code. This assumes that the load factors in the old code have been developed over a long period and represent a good engineering estimate of the required safety. Siu, Parioni, and Lind (1975) used this technique to estimate the weighted average  $\beta$  values in ACI 318-77:

$$\begin{aligned} \text{Flexure } \beta &= 4.2, P_f \approx 1.3 \times 10^{-5} \\ \text{Tied columns } \beta &= 5.22, P_f \approx 2 \times 10^{-7} \\ \text{Shear } \beta &= 3.64, P_f \approx 1.3 \times 10^{-4} \end{aligned}$$

9. The shortcoming of calibration as the sole means of setting the value of  $\beta$  for new criteria is that the level of safety in the current code will generally vary widely from one type of member to

another, as shown by the values listed in the previous paragraph. A uniform value of  $\beta$  for all structural members is more desirable because it will provide a more consistent reliability for the new criteria.

10. An alternative to calibration is to select a probability of failure comparable to the risks people are prepared to accept in other activities. The risks involved in a number of activities are presented in Table C1.

Table C1  
Risk of Death for Various Activities\*

| Activity                                   | Death Rate per Person<br>per year |                      |
|--|-----------------------------------|----------------------|
|  | Those<br>Concerned                | Total<br>Population  |
| Motorcycle racing                          | $5 \times 10^{-4}$                |                      |
| Mountain climbing                          | $5 \times 10^{-3}$                |                      |
| Mining                                     | $7 \times 10^{-4}$                |                      |
| Swimming                                   | $1 \times 10^{-4}$                | $2 \times 10^{-5}$   |
| Automobile travel                          |                                   | $3.6 \times 10^{-4}$ |
| Airplane travel                            | $1 \times 10^{-4}$                |                      |
| Fire in buildings                          |                                   | $2 \times 10^{-5}$   |
| Poisoning                                  |                                   | $1.1 \times 10^{-5}$ |
| Lightning                                  |                                   | $5 \times 10^{-7}$   |
| Vaccinations and inoculations              |                                   | $1 \times 10^{-8}$   |
| Structural collapse<br>during construction | $3 \times 10^{-5}$                |                      |
| All others                                 |                                   | $2 \times 10^{-6}$   |

\* From MacGregor (1976).

This method has the advantage of obtaining a desired degree of reliability for all structural members. This approach is chosen herein.

11. Table C1 suggests that the probability of failure of an RCHS should be about  $10^{-6}$  per year. This corresponds to about  $3 \times 10^{-5}$  during the 30-year life of an RCHS. Assuming normal distributions of  $R$  and  $U$ , the probability of failure of  $3 \times 10^{-5}$  will yield a  $\beta$  value of approximately 4.0 (MacGregor 1976). A value of  $\beta = 4.0$  will be used.

## Derivation of Load Factors and Strength Reduction Factors

### Selection of statistical properties of variables

12. The properties assumed in the calculations are listed in Table C2.

Table C2  
Statistical Distributions Assumed in Calculations

|   | Specified | Mean<br>in situ | Percent | $\sigma$ | V    |
|---|-----------|-----------------|---------|----------|------|
| <u>Material Strengths M</u>               |           |                 |         |          |      |
| Concrete strength                         | 4000 psi  | 3800 psi        | 0.95    | --       | 0.18 |
| Steel yield strength                      | 40 ksi    | 41 ksi          | 1.03    | --       | 0.07 |
| <u>Dimensions E</u>                       |           |                 |         |          |      |
| b-beam, column, in.                       | 12        | 12.05           | 1.004   | 0.3      | 0.05 |
| d-beam, in.                               | 18        | 17.85           | 0.992   | 0.45     | 0.05 |
| h-column, in.                             | 12        | 12.05           | 1.004   | 0.3      | 0.05 |
| A <sub>s</sub> -beam, in. <sup>2</sup>    | 2.00      | 2.00            | 1.00    | --       | 0.06 |
| A <sub>s</sub> -column, in. <sup>2</sup>  | 2.16      | 2.16            | 1.00    | --       | 0.06 |
| <u>Accuracy of Code Equations P</u>       |           |                 |         |          |      |
| M <sub>u</sub> -under-reinforced beams    | --        | --              | 1.06    | --       | 0.04 |
| P <sub>u</sub> -axially loaded columns    | --        | --              | 0.98    | --       | 0.05 |
| V <sub>c</sub> -shear carried by concrete | --        | --              | 1.10    | --       | 0.15 |
| <u>Loadings S</u>                         |           |                 |         |          |      |
| Dead load                                 | --        | --              | 0.9     | --       | 0.14 |
| Live load                                 | --        | --              | 0.8     | --       | 0.30 |
| <u>Structural Analysis E</u>              |           |                 |         |          |      |
| Dead load effects                         | --        | --              | 1.0     | --       | 0.10 |
| Live load effects                         | --        | --              | 1.0     | --       | 0.30 |

### Computation of $\phi$ for flexure of a reinforced concrete beam

13. Design strength R. The assumptions in ACI 318-77 could be used to calculate the beam strength (neglecting  $\phi$ ) as

$$\begin{aligned}
 R = M_u &= A_s f_y \left( d - \frac{a}{2} \right) \\
 &= 2.0 \times 40 \left( 18 - \frac{2.0 \times 40}{2 \times 0.85 \times 4 \times 12} \right) \div 12 \\
 &= 113.5 \text{ kip-ft}
 \end{aligned}$$

14. Mean strength  $R_m$ . The mean strength can be computed using the mean strengths and dimensions in the equation for  $M_u$ .

$$\bar{M}_u = \bar{A}_s \bar{f}_y \left( \bar{d} - \frac{\bar{a}}{2} \right)$$

where

$$\frac{\bar{a}}{2} = \frac{2.0 \times 41}{2 \times 0.85 \times 3.8 \times 12.05} = 1.05$$

$$\bar{M}_u = 2.0 \times 41(17.85 - 1.05) \div 12$$

$$= 114.8 \text{ kip-ft}$$

This value of  $\bar{M}_u$  must be corrected to allow for errors in the equation itself. Thus

$$R_m = \bar{M}_u \times \bar{P} = 114.8 \times 1.06$$

$$= 121.7 \text{ kip-ft}$$

$$\gamma_R = \frac{R_m}{R}$$

(Cl2 bis)

$$= \frac{121.7}{113.5} = 1.07$$

15. Coefficient of variation  $V_R$ . The coefficient of variation

will be calculated in a number of stages

- a.  $V_{a/2}$  - Since  $a/2$  is the product of a number of variables, the coefficient of variation of  $a/2$  can be computed as follows.

$$\begin{aligned} V_{\frac{a}{2}} &= \sqrt{V_{A_s}^2 + V_{f_y}^2 + V_{f_c}^2 + V_b^2} \\ &= \sqrt{0.06^2 + 0.07^2 + 0.18^2 + 0.05^2} \\ &= 0.21 \end{aligned}$$

- b.  $V_{d-\frac{a}{2}}$  - Since  $d - \frac{a}{2}$  is the sum of two variables, it is necessary to compute the standard deviation of this term and then convert that to a coefficient of variation. Thus

$$\sigma_{d-\frac{a}{2}} = \sqrt{\sigma_d^2 + \sigma_{\frac{a}{2}}^2}$$

where

$$\begin{aligned} \sigma_{\frac{a}{2}} &= \left(\frac{\bar{a}}{2}\right) V_{\frac{a}{2}} = 1.05 \times 0.21 \\ &= 0.22 \end{aligned}$$

$$\sigma_{d-\frac{a}{2}} = \sqrt{0.45^2 + 0.22^2} = 0.50$$

and



$$v_{\left(d-\frac{a}{2}\right)} = \frac{\sigma_{\left(d-\frac{a}{2}\right)}}{\bar{d} - \frac{\bar{a}}{2}}$$

$$= \frac{0.50}{17.85 - 1.05} = 0.03$$

c.  $V_{M_u}$  - Because  $M_u$  is calculated as the product of  $A_s$ ,  $f_y$ , and  $(d - a/2)$ ,  $V_{M_u}$  can be calculated as

$$V_{M_u} = \sqrt{V_{A_s}^2 + V_{f_y}^2 + v_{\left(d-\frac{a}{2}\right)}^2}$$

$$= \sqrt{0.06^2 + 0.07^2 + 0.03^2}$$

$$= 0.10$$

d.  $V_R$  - It is necessary to include the effect of the accuracy of the design equation in the coefficient of variation

$$V_R = \sqrt{V_{M_u}^2 + V_p^2}$$

$$= \sqrt{0.10^2 + 0.04^2}$$

$$= 0.10$$

16. Computation of  $\phi$ . According to Equation C16

$$\phi = \gamma_R e^{-\beta \alpha V_R} \quad (C16 \text{ bis})$$

As discussed in paragraph 12,  $\beta = 4.0$ . The term  $\alpha$  will be taken as 0.75 as explained in paragraph 5. Thus

$$\begin{aligned}\phi &= 1.07 e^{-4.0 \times 0.75 \times 0.10} \\ &= 0.78\end{aligned}$$

The value of  $\phi$  for this particular problem is 0.78. This value of  $\phi$  will correspond to the load factors to be developed in paragraph 25. By calculating  $\phi$  values for a range of different properties, a weighted average value can be obtained

Computation of  $\phi$  for an  
axially loaded tied column

17. Design strength  $R$ . The design strength can be calculated by the following equation.

$$\begin{aligned}P_o &= 0.85f'_c(A_g - A_{st}) + (f_y A_{st}) \\ &= 0.85 \times 4 (144 - 2.16) + (40 \times 2.16) \\ &= 568.7 \text{ kips}\end{aligned}$$

18. Mean strength  $R_m$ . Based on the mean strengths and dimensions, the mean axial load capacity is

$$\begin{aligned}\bar{P}_o &= 0.85 \times 3.8 (12.05 \times 12.05 - 2.16) + (41 \times 2.16) \\ &= 550.6 \text{ kips}\end{aligned}$$

Again, this value must be corrected to allow for errors in the equation itself. Thus

$$\begin{aligned}R_m &= \bar{P}_o \times \bar{\phi} = 550.6 \times 0.98 \\ &= 539.6 \text{ kips}\end{aligned}$$

and

$$\gamma_R = \frac{R_m}{R} = \frac{539.6}{568.7}$$

$$= 0.95$$

19. Coefficient of variation  $V_R$ . The strength of the axially loaded column is the sum of the load carried by the concrete  $P_c$  and the steel  $P_s$ . The coefficient of variation of  $P_c$  and  $P_s$  will be evaluated separately.

a.  $V_{P_c}$  and  $\sigma_{P_c}$

$$\begin{aligned} V_{P_c} &= \sqrt{V_{P_c}^2 + V_{P_s}^2 + V_{P_m}^2} \\ &= \sqrt{0.18^2 + 0.00^2 + 0.00^2} \\ &= 0.19 \end{aligned}$$

The standard deviation of the load carried by the concrete is

$$\sigma_{P_c} = V_{P_c} \times \bar{P}_c$$

$$= 0.19 \times 462 = 87.8 \text{ kips}$$

b.  $V_{P_s}$  and  $\sigma_{P_s}$

$$\begin{aligned}
 v_{P_s} &= \sqrt{v_{f_y}^2 + v_{A_{st}}^2} \\
 &= \sqrt{0.07^2 + 0.06^2} \\
 &= 0.09
 \end{aligned}$$

and

$$\sigma_{P_s} = 0.09 \times 88.6 = 8.2 \text{ kips}$$

c.  $v_{P_o}$  and  $\sigma_{P_o}$ . Since  $P_o$  is a sum,  $\sigma$  values must be combined to get  $\sigma_{P_o}$ .

$$\begin{aligned}
 \sigma_{P_o} &= \sqrt{\sigma_{P_c}^2 + \sigma_{P_s}^2} \\
 &= \sqrt{87.8^2 + 8.2^2} = 88.2 \text{ kips}
 \end{aligned}$$

Thus

$$v_{P_o} = \frac{\sigma_{P_o}}{P_o} = \frac{88.2}{550.6} = 0.16$$

Again, this must be adjusted to allow for errors in the equation used to compute  $P_o$

$$v_R = \sqrt{v_{P_o}^2 + v_P^2} = \sqrt{0.16^2 + 0.09^2} = 0.17$$

20. Computation of  $\phi$  . According to Equation C16

$$\begin{aligned}\phi &= \gamma_R e^{-\beta \alpha V_R} \\ &= 0.95 e^{-4.0 \times 0.75 \times 0.17} \quad (\text{C16 bis}) \\ &= 0.57\end{aligned}$$

The value of  $\phi$  for this particular example is 0.57. Before a final value can be chosen, a number of different column sections, including eccentrically loaded columns, must be studied.

Computation of  $\phi$  for shear

21. Design Strength  $R_u$  .

$$\begin{aligned}V_u &= 2 \sqrt{f'_c} \, b d \\ &= 2 \times \sqrt{4000} \times 12 \times \frac{18}{1000} \\ &= 27.3 \text{ kips}\end{aligned}$$

22. Mean Strength  $R_m$  .

$$\begin{aligned}\bar{V}_u &= 2 \times \sqrt{3800} \times 12.05 \times \frac{17.85}{1000} \\ &= 26.5 \text{ kips} \\ R_m &= \bar{V}_u \times \bar{F} = 26.5 \times 1.10 \\ &= 29.1 \text{ kips}\end{aligned}$$

and

$$\gamma_R = \frac{R_m}{R} = \frac{29.2}{27.3} = 1.07$$

23. Coefficient of variation  $V_R$  .

$$\begin{aligned} V_{V_c} &= \sqrt{V_{f_c}^2 + V_b^2 + V_d^2} \\ &= \sqrt{0.18^2 + 0.05^2 + 0.05^2} = 0.19 \end{aligned}$$

$$\begin{aligned} V_R &= \sqrt{V_{V_c}^2 + V_P^2} \\ &= \sqrt{0.19^2 + 0.15^2} = 0.24 \end{aligned}$$

24. Computation of  $\phi$  . Referring again to Equation 16

$$\begin{aligned} \phi &= \gamma_R e^{-\beta \alpha V_R} \quad \text{(C16 bis)} \\ &= 1.07 e^{-4.0 \times 0.75 \times 0.24} \\ &= 0.52 \end{aligned}$$

Computation of load factors for dead and live loads

25. The derivation of  $\lambda$  will be based on Equation C17

$$\lambda = \gamma_u e^{\beta \alpha U} \quad \text{(C17 bis)}$$

But,  $U = D + L$  where both  $D$  and  $L$  are separate variables. Because the coefficient of variation of  $L$  is much smaller than that of

L, it is desirable to separate these.

Since

$$U_m = U \gamma_u \quad (C13 \text{ bis})$$

Equation C17 can be rewritten as

$$\gamma U = U_m e^{\beta \alpha V_u} \quad (C18)$$

Using the first two terms of the series expansion for  $e^x$  gives

$$\begin{aligned} U_m e^{\beta \alpha V_u} &= (D_m + L_m)(1 + \beta \alpha V_u) \\ &= (D_m + L_m) \left( 1 + \frac{\beta \alpha \sqrt{D_m^2 V_D^2 + L_m^2 V_L^2}}{D_m + L_m} \right) \\ &\approx (D_m + L_m) \left( 1 + \frac{\beta \alpha^2 D_m V_D + \beta \alpha^2 L_m V_L}{D_m + L_m} \right) \end{aligned}$$

or

$$U_m e^{\beta \alpha V_u} \approx D_m \left( 1 + \beta \alpha^2 V_D \right) + L_m \left( 1 + \beta \alpha^2 V_L \right) \quad (C19)$$

The parenthetical terms in Equation C19 can be rewritten in exponential form for consistency, giving separate load factors for D and L.

$$\lambda_D = \gamma_D e^{\beta \alpha^2 V_D} \quad (C20)$$

$$\lambda_L = \gamma_L e^{\beta \alpha^2 V_L} \quad (C21)$$

26. Derive  $\lambda_D$ .

$$\lambda_D = \gamma_D e^{\beta \alpha^2 V_D} \quad (\text{C20 bis})$$

The term  $V_D$  is affected by variations in the load  $V_{SD}$  and variations due to standard analysis  $V_{ED}$ . Thus

$$\begin{aligned} V_D &= \sqrt{V_{SD}^2 + V_{ED}^2} \\ &= \sqrt{0.15^2 + 0.10^2} = 0.18 \end{aligned}$$

Therefore

$$\begin{aligned} \lambda_D &= 0.9 e^{4.0 \times 0.75^2 \times 0.18} \\ &= 1.35 \end{aligned}$$

27. Derive  $\lambda_L$ .

$$\lambda_L = \gamma_L e^{\beta \alpha^2 V_L} \quad (\text{C21 bis})$$

where

$$\begin{aligned} V_L &= \sqrt{V_{SL}^2 + V_{EL}^2} \\ &= \sqrt{0.3^2 + 0.2^2} = 0.36 \end{aligned}$$

$$\begin{aligned} \lambda_L &= 0.8 e^{4.0 \times 0.75^2 \times 0.36} \\ &= 1.80 \end{aligned}$$



28. Other load combinations. The above derivation was limited to the combination of dead plus live loads. Similar analyses are required for other loading combinations.

#### Discussion

29. Load factors and strength reduction factors derived from these particular examples compared to those specified in Part II are summarized in the following tabulation.

| Factor                  | Calculated<br>Values | Calculated<br>Values $\times 1.11$ | Specified<br>Values for<br>Design |
|-------------------------|----------------------|------------------------------------|-----------------------------------|
| Dead load               | 1.35                 | 1.50                               | 1.50                              |
| Live load               | 1.80                 | 2.00                               | 1.90                              |
| $\phi$ for flexure      | 0.78                 | 0.87                               | 0.90*                             |
| $\phi$ for tied columns | 0.57                 | 0.63                               | 0.70*                             |
| $\phi$ for shear        | 0.52                 | 0.58                               | 0.85*                             |

\* Same as ACI 318-77 (ACI 1977a).

The calculated values, except for shear, are very close to those specified in Part II. The strength reduction factor for shear given in ACI 318-77 appears to be inadequate; the 0.85 value is insufficient to account for the large coefficient of variation in shear resistance. This deficiency has also been noted by other investigators (Illingwood 1978).

30. Much more extensive study of various sizes of members, reinforcing ratios, and live to dead load ratios is required before final load factors and  $\phi$  factors can be determined. However, the final values are expected to be similar to the ones derived in this appendix.

#### Shortcomings of First-Order Probabilistic Procedure

31. According to MacGregor (1976), there are four major shortcomings of the first-order probabilistic procedure used to calculate load and strength reduction factors.

- a. The procedure is only as good as the data used in the solution. This is a problem to all procedures. Statistical data of the type required are not widely available.
- b. The procedure assumes that specific probabilities of failure can be evaluated. Since this calculation depends on a knowledge of the extreme ranges of the strength distributions that are not adequately known, the computed probabilities of failure could differ from the actual values by as much as a factor of 10.
- c. Failure of the structure is assumed to occur when one cross section or element reaches its capacity. For a weakest-link structure such as a truss, failure will occur when the weakest of many elements is overloaded. For a ductile indeterminate structure, loads will be distributed from section to section before the entire structure fails. The solution will tend to overestimate the safety of the first case and underestimate the second.
- d. Only failure by known overloads has been considered. Causes of failure such as gross error, fire, or explosion have not been considered.

32. In spite of these shortcomings, the first-order probabilistic procedure used in this appendix provides a rational procedure for estimating safety factors and should be used to derive load and strength reduction factors for RCHS.

#### Recommended Future Studies

33. It is envisaged that the following efforts are required for a rational procedure for derivation of load and strength reduction factors can be realized:

- a. Collect data on statistical distribution. Extensive data of the type summarized in Table 1 must be collected for each component affecting the strength of RCHS. In addition, data must be obtained on the magnitude of about typical dead, live, and wind loads, water pressures in RCHS, as well as typical material strengths and steel percentages, so that a rational design can be used for the most common cases.
- b. Determine theoretical and design procedures. Procedures for calculating the strength of RCHS must be selected and compared with existing design equations should be used.

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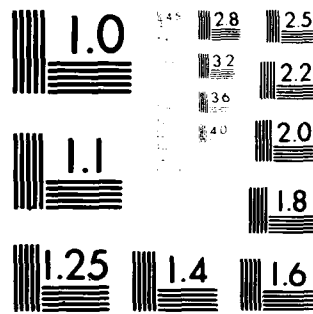
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theoretical equations should be as definitive as necessary to accurately estimate the true member strength.

- c. Calculate  $\gamma_R$  and  $V_R$  for various structural actions. Values of  $\gamma_R$  and  $V_R$  must be calculated for flexure, column cross sections in combined bending and axial load, slender columns, shear, bond, and deflections, and possibly for cracking. For some members these terms can be computed using direct statistical methods similar to those outlined in this appendix. For other members such as column cross sections or slender columns, the interaction of the variables is more complex and a Monte-Carlo simulation technique must be employed to estimate  $\gamma_R$  and  $V_R$ .

APPENDIX D: DERIVATION OF MAXIMUM TENSION REINFORCEMENT  
RATIO FOR FLEXURAL MEMBERS

1. With reference to Figure D1, the equilibrium between external and internal bending moments  $M$  at working stress, can be expressed as

$$M = \frac{1}{2} f_c k j b d^2 \quad (D1)$$

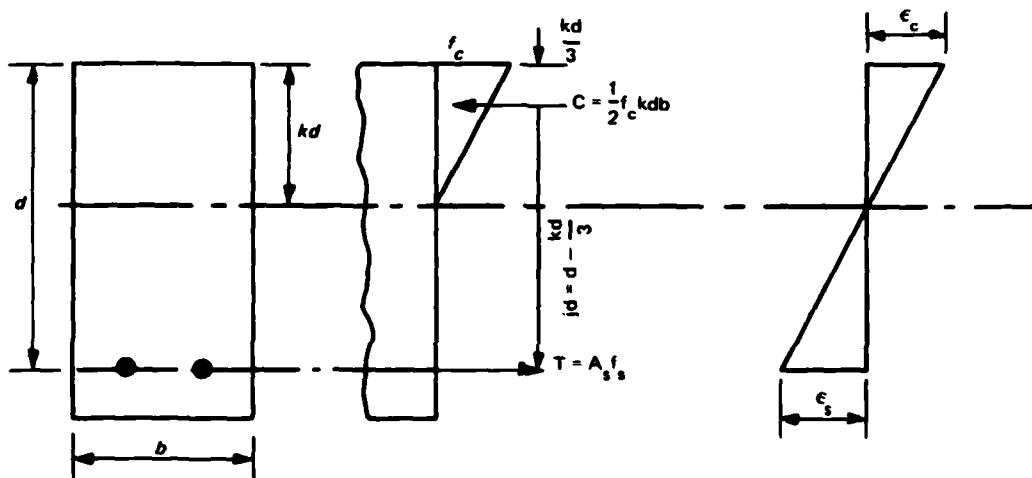


Figure D1. Stress and strain distribution,  
working-stress design

The minimum  $d$  required for working-stress design (WSD) can be obtained from Equation D1

$$d = \sqrt{\frac{2M}{f_c k j b}} \quad (D2)$$

where  $k$  and  $j$  are the values for the balanced WSD stress condition.

2. From Figure D2, the factored moment strength  $M_u$  can be written as

$$M_u = \phi \left[ A_s f_y \left( d - \frac{a}{2} \right) \right] \quad (D3)$$

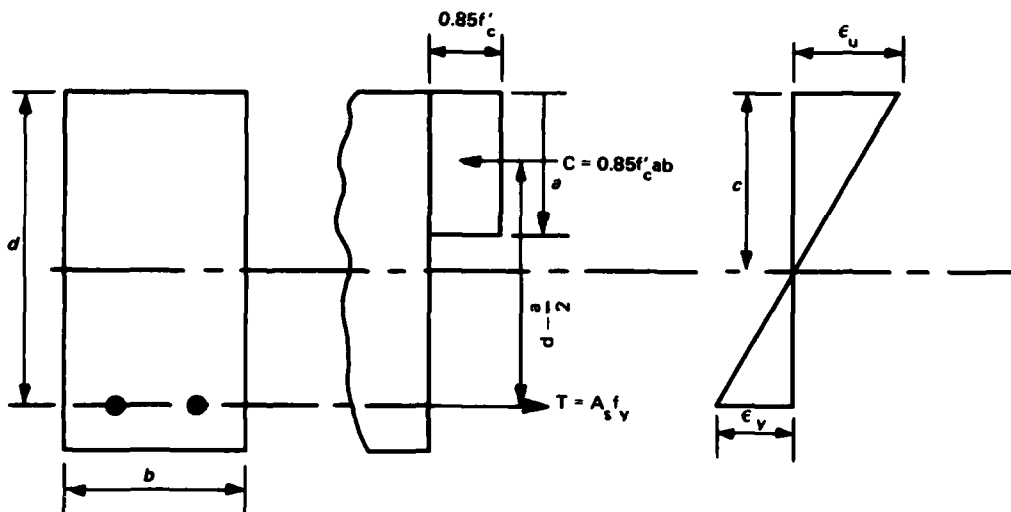


Figure D2. Stress and strain distribution, strength design

and

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{\rho f_y d}{0.85 f'_c} \quad (D4)$$

By substituting the value of  $a$  obtained from Equation D4 and substituting  $\rho b d$  for  $A_s$  into Equation D3

$$M_u = \phi \rho f_y b d^2 \left( 1 - 0.59 \rho \frac{f_y}{f'_c} \right) \quad (D5)$$

Thus

$$d = \sqrt{\frac{M_u}{\phi \rho f_y \left( 1 - 0.59 \rho \frac{f_y}{f'_c} \right) b}} \quad (D6)$$

3. For an equivalent design, Equation D2 should be equal to Equation D6, and therefore

$$\frac{2M}{f'_c k j b} = \frac{M_u}{\phi \rho f_y \left(1 - 0.59 \rho \frac{f_y}{f'_c}\right) b} \quad (D7)$$

For hydraulic structures

$$M_u \leq 1.9 M \quad (D8)$$

$$f'_c = 0.35 f'_c \quad (D9)$$

$$\phi = 0.9 \quad (D10)$$

$$f_y = 40 \text{ ksi} \quad (D11)$$

Substituting Equations D8-D11 into Equation D7

$$72\rho \left(1 - 0.59\rho \frac{40}{f'_c}\right) = 0.67 f'_c k j \quad (D12)$$

The tension reinforcement ratio for equivalent design  $\rho$  can be obtained from Equation D12 for given  $f'_c$ ,  $k$ , and  $j$ . Table D1 shows the  $\rho$  and  $\rho/\rho_b$  values for various  $f'_c$ . It can be seen that  $\rho/\rho_b$  varies from 0.23 to 0.29. The value of 0.25 is recommended for design of hydraulic structures. For conduits or culverts, the allowable concrete stress  $f'_c$  is  $0.45 f'_c$ . Substituting this value together with Equations D8, D10, and D11 into Equation D7

$$72\rho \left(1 - 0.59\rho \frac{40}{f'_c}\right) = 0.86 f'_c k j \quad (D13)$$

The  $\rho$  values for conduits or culverts for various  $f'_c$  are given in Table D2. The  $\rho/\rho_b$  varies from 0.36 to 0.442. The recommended value of  $\rho$  for conduits or culverts is  $0.375\rho_b$ .



Table D1  
Tension Reinforcement Ratio for Equivalent  
Design of Hydraulic Structures

| $f'_c$<br>ksi | $0.35f'_c$<br>ksi | $k^*$ | $j^*$ | $\rho$ | $\rho_b^{**}$ | $\rho/\rho_b$ |
|---------------|-------------------|-------|-------|--------|---------------|---------------|
| 3             | 1.050             | 0.326 | 0.891 | 0.0086 | 0.0371        | 0.23          |
| 4             | 1.400             | 0.359 | 0.880 | 0.0128 | 0.0490        | 0.26          |
| 5             | 1.750             | 0.383 | 0.872 | 0.0169 | 0.0582        | 0.29          |

\* Obtained from American Concrete Institute (1965).

\*\* Obtained from American Concrete Institute (1977b):

$$\rho_b = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{87,000}{87,000 + f_y}$$

Table D2  
Tension Reinforcement for Equivalent Design  
of Conduits or Culverts

| $f'_c$<br>ksi | $0.45f'_c$<br>ksi | $k^*$ | $j^*$ | $\rho$ | $\rho_b^{**}$ | $\rho/\rho_b$ |
|---------------|-------------------|-------|-------|--------|---------------|---------------|
| 3             | 1.350             | 0.383 | 0.872 | 0.0134 | 0.0371        | 0.360         |
| 4             | 1.800             | 0.419 | 0.860 | 0.0195 | 0.0495        | 0.393         |
| 5             | 2.250             | 0.444 | 0.852 | 0.0257 | 0.0582        | 0.442         |

\* Obtained from American Concrete Institute (1965).

\*\* Obtained from American Concrete Institute (1977b):

$$\rho_b = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{87,000}{87,000 + f_y}$$

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Liu, Tony C

Strength design of reinforced concrete hydraulic structures; Report 1: Preliminary strength design criteria / by Tony C. Liu. Vicksburg, Miss. : U. S. Waterways Experiment Station ; Springfield, Va. : available from National Technical Information Service, 1980.

27, [70] p. : ill. ; 27 cm. (Technical report - U. S. Army Engineer Waterways Experiment Station ; SL-80-4, Report 1)

Prepared for Office, Chief of Engineers, U. S. Army, Washington, D. C., under CWIS 31623.

References: p. 26-27.

1. Concrete strength. 2. Concrete structures. 3. Hydraulic structures. 4. Reinforced concrete. I. United States. Army. Corps of Engineers. II. Series: United States. Waterways Experiment Station, Vicksburg, Miss. Technical report ; SL-80-4, Report 1.

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